1.3
Notes

SOLVING EQUATIONS

OBJECTIVES: 1) Determine the domain of a variable.
2) Solve literal equations.
2) Solve quadratic equations.

WHAT IS "LEGAL"?

1) Adding/Subtracting same quantity on both sides of an equation produces equiv. equation.
2) Multiplying/Dividing both sides by the same nonzero quantity produces equiv. equation.
3) Simplifying an expression on either side of an equation produces an equiv. equation.

Seem easy? Well it's very easy to make algebraic mistakes. We'll talk about common ones in class.

1. $\dfrac{2}{2x+5}+\dfrac{3}{2x-5}=\dfrac{10x+5}{4x^2-25}$

$2(2x-5)+3(2x+5)=10x+5$

$4x-10+6x+15=10x+5$

$10x+5=10x+5$

$\mathbb{R}$ $x\neq -5/2, 5/2$

$(-\infty,-5/2)\cup(-5/2,5/2)\cup(5/2,\infty)$

DOMAIN Find the domain using interval notation.

2. $\dfrac{x^2+2x+1}{x^2-9}$

$x\neq 3,-3$

$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

3. $\dfrac{x-5}{\sqrt{2-x}}$

$x\neq 2$ $2-x>0$

$x<2$

$(-\infty,2)$

SOLVING LITERAL EQUATIONS

1) ELIMINATE "INTENDED VARIABLE" FROM THE DENOMINATOR.
2) GET ALL "INTENDED VARIABLES" TO ONE SIDE
3) FACTOR OUT VARIABLE
4) DIVIDE

4. Solve for a: $x(c-ay)=ae+f$

$xc-axy=ae+f$

$xc-f=ae+axy$

$xc-f=a(e+xy)$

$\boxed{a=\dfrac{xc-f}{e+xy}}$

5. Solve for b: $a=\dfrac{b}{a+b}$

$a(a+b)=b$

$a^2+ab=b$

$a^2=b-ab$

$a^2=b(1-a)$

$\boxed{b=\dfrac{a^2}{1-a}}$

6. Solve for x: $\dfrac{ax+b}{c-dx}=e$

$ax+b=e(c-dx)$

$ax+b=ec-edx$

$ax+edx=ec-b$

$x(a+ed)=ec-b$

$\boxed{x=\dfrac{ec-b}{a+ed}}$

SOLVING QUADRATIC EQUATIONS

Solve by factoring.

7. $3x^2 + 13x = 10$

$(3x-2)(x+5)=0$

$x = 2/3 \qquad x=-5$

8. $x(x+1) = 182$

$x^2+x-182=0$

$(x+14)(x-13)=0$

$\boxed{x=-14 \quad x=13}$

9. $x^2 + \left(2\sqrt{7}\right)x + 7 = 0$

$(x+\sqrt{7})^2=0$

$\boxed{x=-\sqrt{7}}$

> **METHODS:**
> 1) SQUARE ROOTS
> 2) FACTORING
> 3) QUADRATIC FORMULA
> 4) COMPLETE THE SQUARE

Use square roots:

10. $(x+2)^2 - 72 = 0$

$(x+2)^2 = 72$

$x+2 = \pm 6\sqrt{2}$

$\boxed{x = -2 \pm 6\sqrt{2}}$

Use quadratic formula:

11. $2x^2 - 10 = -\sqrt{2}x$

$2x^2 + \sqrt{2}x - 10 = 0$

$$x = \frac{-\sqrt{2} \pm \sqrt{\sqrt{2}^2 - 4(2)(-10)}}{2(2)} = \frac{-\sqrt{2} \pm \sqrt{2+80}}{4}$$

$$= \boxed{\frac{-\sqrt{2} + \sqrt{82}}{4}}$$

DERIVING THE QUADRATIC FORMULA BY COMPLETING THE SQUARE

Solve by completing the square: $2x^2 - 4x \overset{-3}{\cancel{+3}} = 0$.

$2x^2 - 4x = 3$

$2(x^2 - 2x) = 3$

$2(x^2 - 2x + 1) = 3 + 2$

$2(x-1)^2 = 5$

$(x-1)^2 = 5/2$

$x - 1 = \pm\sqrt{5/2}$

$\boxed{x = 1 \pm \sqrt{5/2}}$

> **COMPLETING THE SQUARE**
> 1.) Move the constant to the side with y
> 2.) Factor out the quadratic coefficient
> 3.) Take half of the linear coefficient and square it. Add this number inside the parentheses. **BALANCE THE EQUATION!**
> 4.) Factor the quantity in parentheses.

Use the "complete the square method" to derive the formula to the solution to $ax^2 + bx + c = 0$.

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

$a\left(x^2 + \frac{b}{a}x\right) = -c$

$\frac{b}{a} \Rightarrow \frac{b}{2a} \Rightarrow \frac{b^2}{4a^2}$

$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + \frac{b^2}{4a}$

$a\left(x + \frac{b}{2a}\right)^2 = -c + \frac{b^2}{4a} = \frac{-4ac + b^2}{4a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

> **DO NOT MEMORIZE THIS! THIS IS A PROCESS YOU NEED TO UNDERSTAND THROUGH PRACTICE!**