1.3
Notes

TECHNIQUES FOR FINDING LIMITS

OBJECTIVES: 1) Use theorems that simplify problems involving limits.
2) Use the Sandwich/Squeeze Theorem to find a limit.

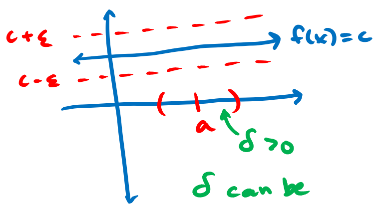
It would be an insane amount of work to verify every limit by means of our precise definition of a limit. Rather than continuing to use the delta/epsilon definition to verify limits, we will now rely on some theorems (that have already been proven) that will simplify our task of finding limits.

So let's piggy back off these theorems:

1) $\lim_{x \to a} c = c$ "The limit of a constant is the constant."

For any ε choose, choose any δ.

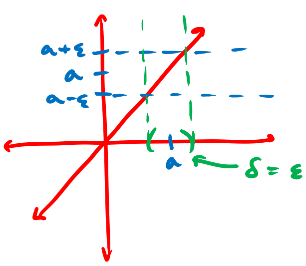
So if $0 < |x-a| < \delta$, then $|c-c| = 0 < \varepsilon$.



δ can be any positive real.

2) $\lim_{x \to a} x = a$ For any ε, choose $\delta = \varepsilon$.

If $0 < |x-a| < \delta$, then $|x-a| < \varepsilon$ b/c $\delta = \varepsilon$



PROPERTIES OF LIMITS

If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both exist, then:

1) $\lim_{x \to a}\left[f(x)+g(x)\right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

The limit of a sum is the sum of the limits.

2) $\lim_{x \to a}\left[f(x) \cdot g(x)\right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

The limit of a product is the product of the limits.

3) $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$

The limit of a quotient is the quotient of the limits, provided the limit in the denominator is not zero.

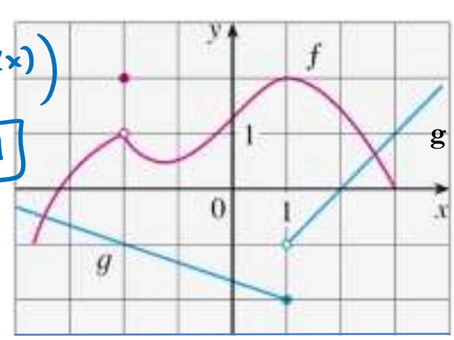
4) $\lim_{x \to a}\left[c f(x)\right] = c\left[\lim_{x \to a} f(x)\right]$ for any number c

The limit of a constant times a function is the constant times the limit of the function.

5) $\lim_{x \to a}\left[f(x)-g(x)\right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

The limit of a difference is the difference of the limits.

Use the graph to answer the following questions.



a. $\lim_{x \to -2}\left[f(x)+5g(x)\right] = \lim_{x \to -2} f(x) + 5\left(\lim_{x \to -2} g(x)\right)$

$1 + 5(-1) = \boxed{-4}$

b. $\lim_{x \to 1^+}\left[f(x)g(x)\right] = \left[\lim_{x \to 1^+} f(x)\right]\left[\lim_{x \to 1^+} g(x)\right]$

$(2)(-1) = \boxed{-2}$

c. $\lim_{x \to 3^-}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to 3^-} f(x)}{\lim_{x \to 3^-} g(x)} = \frac{0}{1} = \boxed{0}$

d. Does $f(-2) = \lim_{x \to -2} f(x)$**?** NOPE.

$2 \neq 1$

EXAMPLES: Find the following limits:

ADDITIONAL PROPERTIES

1) $\lim\limits_{x\to a} x^n = a^n$

2) $\lim\limits_{x\to a}\left[f(x)\right]^n = \left[\lim\limits_{x\to a} f(x)\right]^n$

3) $\lim\limits_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$

4) $\lim\limits_{x\to a}\left(\sqrt[n]{x}\right)^m = \left(\sqrt[n]{a}\right)^m$ or $\lim\limits_{x\to a}(x)^{m/n} = (a)^{m/n}$

5) $\lim\limits_{x\to a}\left(\sqrt[n]{f(x)}\right) = \sqrt[n]{\lim\limits_{x\to a} f(x)}$

1) $\lim\limits_{x\to 3}(2x-3)^4 = \boxed{81}$

$\left[\lim\limits_{x\to 3} 2x-3\right]^4$

$(3)^4 = 81$

2) $\lim\limits_{x\to 3}\sqrt[3]{3x^2-4x+9} = \boxed{2\sqrt[3]{3}}$

$\sqrt[3]{\lim\limits_{x\to 3} 3x^2-4x+9}$

$\sqrt[3]{27-12+9} = \sqrt[3]{24} = 2\sqrt[3]{3}$

3) $\lim\limits_{x\to 8}\dfrac{x^{\frac{2}{3}}+3\sqrt{x}}{4-\dfrac{16}{x}} = \boxed{2+3\sqrt{2}}$

$\dfrac{8^{2/3}+3\sqrt{8}}{4-16/8} = \dfrac{(2^3)^{2/3}+3\cdot 2\sqrt{2}}{4-2}$

$= \dfrac{2^2+6\sqrt{2}}{2} = 2+3\sqrt{2}$

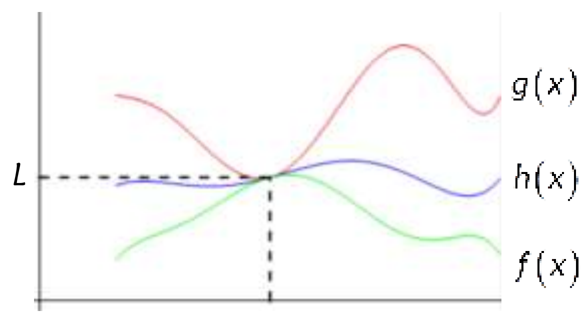
4) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{x} \cdot \dfrac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$

$\dfrac{x^2+4-4}{x\left(\sqrt{x^2+4}+2\right)} = \dfrac{x^2}{x\left(\sqrt{x^2+4}+2\right)} = \dfrac{x}{\sqrt{x^2+4}+2}$

$\lim\limits_{x\to 0}\dfrac{x}{\sqrt{x^2+4}+2} = \dfrac{0}{4} = \boxed{0}$

SANDWICH/SQUEEZE THEOREM: Suppose $f(x)\le h(x)\le g(x)$ for every x in an open interval containing a, except possibly at a. If $\lim\limits_{x\to a} f(x) = L = \lim\limits_{x\to a} g(x)$, then $\lim\limits_{x\to a} h(x) = L$.

Let's consider a problem: Find $\lim\limits_{x\to 0} x^2\cos\left(\dfrac{1}{x}\right)$.



$\left(-1 \le \cos\left(\frac{1}{x}\right) \le 1\right) x^2$

$-x^2 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$

$\lim\limits_{x\to 0} -x^2 \le \lim\limits_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \le \lim\limits_{x\to 0} x^2$

$0 \le \lim\limits_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \le 0$

$\boxed{\lim\limits_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = 0}$