1.4
Notes

LIMITS INVOLVING INFINITY

OBJECTIVES: 1) Find limits at a vertical asymptote.
2) Use two different techniques for evaluating limits at infinity.

FINDING LIMITS AT A VERTICAL ASYMPTOTE:

1) $\lim\limits_{x\to -3} \dfrac{-2}{x+3}$

$\lim\limits_{x\to -3^-} \dfrac{-2}{x+3} = \dfrac{-2}{-0} = +\infty$

$\lim\limits_{x\to -3^+} \dfrac{-2}{x+3} = \dfrac{-2}{+0} = -\infty$

$\boxed{\lim\limits_{x\to -3} \dfrac{-2}{x+3} = \text{DNE}}$

2) $\lim\limits_{x\to 0} \dfrac{1}{x^2}$

$\lim\limits_{x\to 0^-} \dfrac{1}{0^+} = +\infty$

$\lim\limits_{x\to 0^+} \dfrac{1}{0^+} = +\infty$

$\lim\limits_{x\to 0} \dfrac{1}{x^2} = \text{DNE}, +\infty$

LIMITS AT INFINITY:

If k is a positive rational number and c is any real number, then

$$\lim_{x\to\infty} \frac{c}{x^k} = 0 \quad \text{and} \quad \lim_{x\to -\infty} \frac{c}{x^k} = 0$$

(provided that x^k is always defined).

END BEHAVIOR LIMITS: TWO TECHNIQUES

End behavior describes how the function "acts" as $x \to \pm\infty$.

1) $\lim\limits_{x\to\infty}\left(x^3 - 3x^2 + 7x + 2\right)$

$\lim\limits_{x\to\infty} x^3\left(1 - \dfrac{3}{x} + \dfrac{7}{x^2} + \dfrac{2}{x^3}\right) =$

$\infty(1 - 0 + 0 + 0)$

$\lim\limits_{x\to\infty} x^3 - 3x^2 + 7x + 2 = +\infty$

2) $\lim\limits_{x\to -\infty}\left(x^3 - 3x^2 + 7x + 2\right)$

$\lim\limits_{x\to -\infty} f(x) = -\infty$

3) $\lim\limits_{x\to\infty}\left(-7x^4 + 2x^2 + x - 5\right)$

$\lim\limits_{x\to\infty} f(x) = -\infty$

FUNCTION HIERARCHY:

The progression for determining which functions dominate (take over) is

$$\frac{1}{x} < \log x < x < x^2 < x^3 < \ldots < 2^x < 3^x < \ldots < x^x$$

4) $\lim_{x \to \infty} \left(\frac{x^3 - 1}{2x^2 + 7x + 500} \right) = \infty$

5) $\lim_{x \to -\infty} \left(\frac{2x^2 - 5}{3x^2 + x + 2} \right) = \frac{2}{3}$

6) $\lim_{x \to \infty} \left(\frac{x^3 - 6}{2x^4 - 5x} \right) = 0$

7) $\lim_{x \to -\infty} \left(\frac{x^3 - 3x^2 + 7x + 2}{-3x^3 - x + 2} \right) = -\frac{1}{3}$

8) $\lim_{x \to -\infty} \frac{\sqrt{9x^2 + 2}}{4x + 3} = \frac{-3}{4}$

9) $\lim_{x \to \infty} \left(\frac{2x^4 - x^2 + 8x}{-5x^4 + 7} \right) = -\frac{2}{5}$