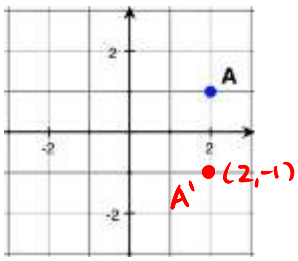
SYMMETRY AND GRAPHS

OBJECTIVES:
1) Use symmetry to aid in graphing.
2) Use algebraic tests to determine the type of symmetry of graphs of equations.
3) Complete the square to write equations of circles in standard form.

REFLECTIONS

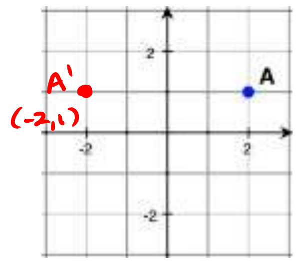
Sketch the reflection of point **A** about:

The x-axis



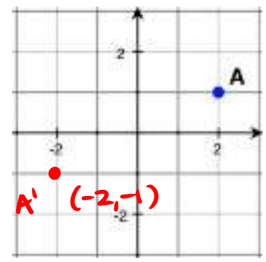
(x, y) becomes

The y-axis



(x, y) becomes

The origin (x and y)



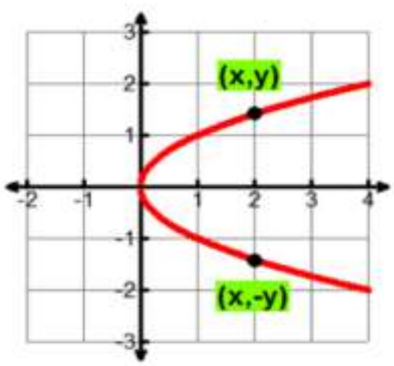
(x, y) becomes

Ex) The graph of $y = f(x)$ contains the point (2, –3). What point must lie on the reflected graph if the graph is reflected a) about the x-axis? b) about the y-axis? c) about the origin?

a) (2,3) b) (-2,-3) c) (-2,3)

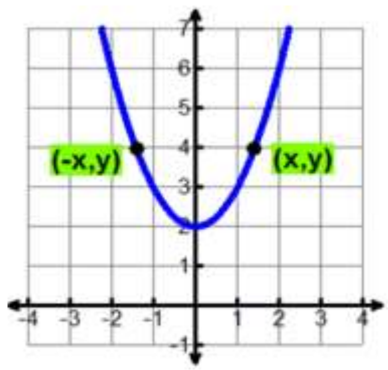
SYMMETRY

Symmetric with respect to the x-axis



TEST: Plug in –y.

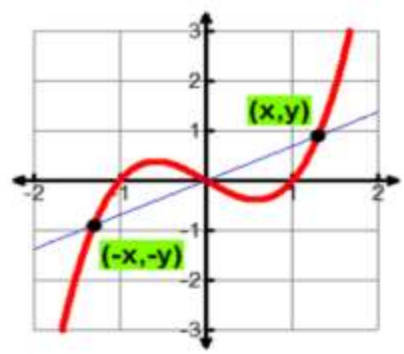
Symmetric with respect to the y-axis



EVEN

TEST: Plug in –x.

Symmetric about the origin



ODD

TEST: Plug in –x and –y.

Ex) Determine the symmetry of the graph of each equation.

a. $y^2 = x + 4$

$(-y)^2 = x + 4$

$y^2 = x + 4$

sym. w.r.t. x-axis

b. $y = x^2 - 2$

$y = (-x)^2 - 2$

$y = x^2 - 2$

sym. w.r.t. y-axis

c. $y = x^3 - 4x$

$-y = (-x)^3 - 4(-x)$

$-y = -x^3 + 4x$

$y = x^3 - 4x$

sym. wrt origin

Circle!

d. $x^2 + y^2 = 4$

$(-x)^2 + y^2 = 4$

$x^2 + y^2 = 4$

$x^2 + (-y)^2 = 4$

$x^2 + y^2 = 4$

$(-x)^2 + (-y)^2 = 4$

$x^2 + y^2 = 4$

sym. w.r.t x-axis y-axis origin

CALCULATOR EXAMPLE

$y = \dfrac{1}{(x^3 - x)}$ Standard Viewing Rectangle [-2, 2, 1] by [-10, 10, 1]

sym. wrt. origin

$-y = \dfrac{1}{(-x)^3 - (-x)}$

$-y = \dfrac{1}{-x^3 + x}$ $\boxed{y = \dfrac{1}{x^3 - x}}$

CIRCLES: Equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$ where C(h,k) and r is the radius.

1. Find the equation and graph the circle with diameter endpoints (-4,1) and (0,5).

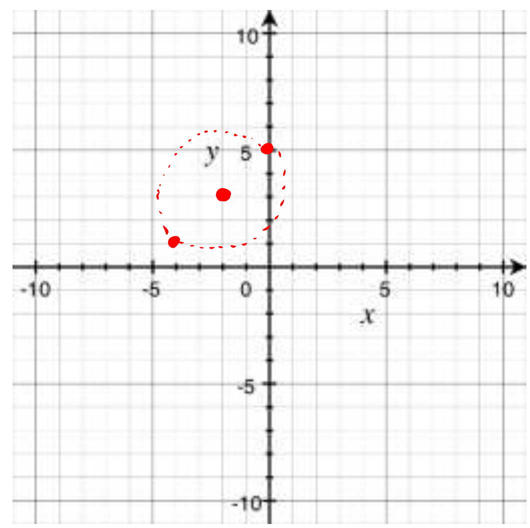
diameter $(-4,1)(0,5)$

$d = \sqrt{(-4-0)^2 + (1-5)^2} = \sqrt{32} = 4\sqrt{2}$ radius $= 2\sqrt{2}$

midpt $(-4,1)(0,5)$ center: $(-2,3)$

$\left(\frac{-4}{2}, \frac{6}{2}\right) \Rightarrow (-2,3)$

$(x+2)^2 + (y-3)^2 = (2\sqrt{2})^2 = 8$



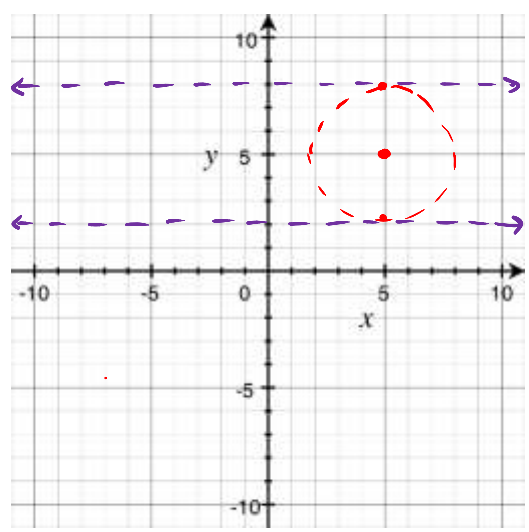
2. A circle is tangent to the lines y = 2 and y=8, and the center goes through the line x=5. Find the equation for the circle.

center: $(5, y)$ y is midway between $y=2$ & $y=8$

$y=5$

$C(5,5)$ radius $= 3$

$\boxed{(x-5)^2 + (y-5)^2 = 9}$



GRAPHING ON A CALCULATOR:
1) Solve for y (There should be two functions: + and –.)
2) Set window with Zoom 5:Zsquare

COMPLETING THE SQUARE: Rewriting quadratics into factored form

3. $16x^2 - 8x + 16y^2 - 64y - 15 = 0$

$16(x^2 - \frac{1}{2}x) + 16(y^2 - 4y) = 15$

$16(x^2 - \frac{1}{2}x + \frac{1}{16}) + 16(y^2 - 4y + 4) = 15 + 1 + 64$

$16(x - \frac{1}{4})^2 + 16(y-2)^2 = 80$

$\boxed{(x - \frac{1}{4})^2 + (y-2)^2 = 5}$

COMPLETING THE SQUARE

1.) Move the constant to the side with y

2.) Factor out the quadratic coefficient

3.) Take $\frac{1}{2}$ of the linear coefficient and square it. Add this number inside the parentheses. **BALANCE THE EQUATION!**

4.) Factor the quantity in parentheses.

4. $3x^2 - 18 + 3y^2 + 6\sqrt{2}y = 0$

$3(x^2) + 3(y^2 + 2\sqrt{2}y) = 18$

$3(x-0)^2 + 3(y^2 + 2\sqrt{2}y + 2) = 18 + 6$

$3(x-0)^2 + 3(y + \sqrt{2})^2 = 24$

$\boxed{(x-0)^2 + (y + \sqrt{2})^2 = 8}$

5. $ax^2 - bx + 2 = 0$

$a(x^2 - \frac{b}{a}x) = -2$

$a(x^2 - \frac{b}{a}x + \frac{b^2}{4a^2}) = -2 + \frac{b^2}{4a}$

$a(x - \frac{b}{2a})^2 = \frac{-8a + b^2}{4a}$

$(x - \frac{b}{2a})^2 = \frac{-8a + b^2}{4a^2}$

$x - \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 8a}{4a^2}}$

$\boxed{x = \frac{b \pm \sqrt{b^2 - 8a}}{2a}}$