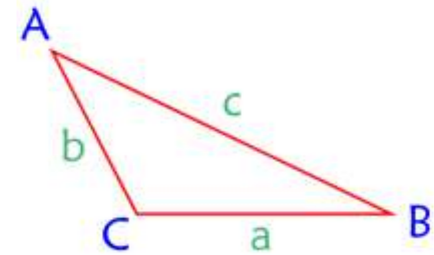
(PART 1) THE LAW OF SINES AND LAW OF COSINES

OBJECTIVES: 1) Use the law of sines and law of cosines to solve a triangle.

THE LAW OF SINES:

In any triangle, the ratio of the sine of an angle to the length of its opposite side is constant.

$$\frac{\sin A}{A}=\frac{\sin B}{B}=\frac{\sin C}{C} \qquad \frac{A}{\sin A}=\frac{B}{\sin B}=\frac{C}{\sin C}$$



A Quick Proof:

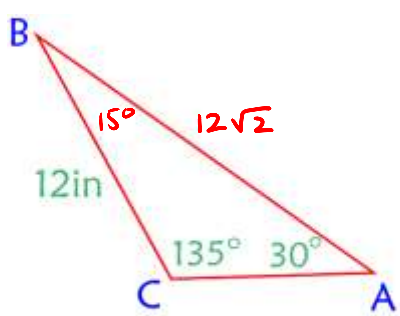
$$\frac{2}{abc}\cdot\left(\frac{1}{2}ab\sin C=\frac{1}{2}ac\sin B=\frac{1}{2}cb\sin A\right)$$

$$\frac{\sin C}{c}=\frac{\sin B}{b}=\frac{\sin A}{a}$$

EXAMPLES

Use the law of sines to solve the triangle.

1)



$$\frac{\sin 135^\circ}{c}=\frac{\sin 30^\circ}{12}$$

$$c=\frac{12\sin 135^\circ}{\sin 30^\circ}$$

$$c=\frac{12\frac{\sqrt{2}}{2}}{\frac{1}{2}}=\boxed{12\sqrt{2}}$$

$$\frac{\sin 15^\circ}{b}=\frac{\sin 30^\circ}{12}$$

$$b=\frac{12\sin 15^\circ}{\sin 30^\circ}$$

$$b=\frac{12\sqrt{\frac{1-\cos 30^\circ}{2}}}{\frac{1}{2}}$$

$$b=24\sqrt{\frac{1-\sqrt{3}/2}{2}}$$

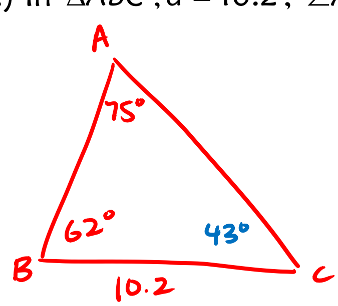
$$b=24\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$b=\frac{24\sqrt{2-\sqrt{3}}}{2}$$

$$b=12\sqrt{2-\sqrt{3}}$$

$$b\approx 6.2$$

2) In ΔABC, $a=10.2$, $\angle A=75^\circ$, and $\angle B=62^\circ$.



$$\frac{\sin 75^\circ}{10.2}=\frac{\sin 62^\circ}{b}$$

$$b=\frac{10.2\sin 62^\circ}{\sin 75^\circ} \qquad b\approx 9.3$$

$$\frac{\sin 43^\circ}{c}=\frac{\sin 75^\circ}{10.2}$$

$$c=\frac{10.2\sin 43^\circ}{\sin 75^\circ} \qquad c\approx 7.2$$

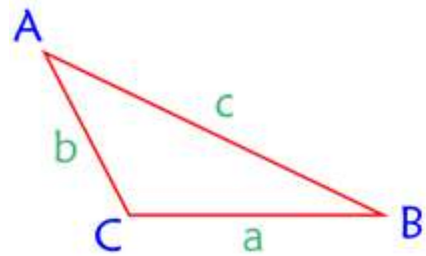
THE LAW OF COSINES:

In any triangle, the square of the length of any side equals the sum of the squares of the lenghts of the other two sides minus twice the product of the lenghts of those other two sides times the cosine of their included angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

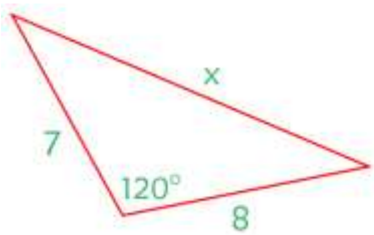
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



FINDING A LENGTH

3)



$$x^2 = 7^2 + 8^2 - 2(7)(8)\cos 120^\circ$$

$$x^2 = 49 + 64 - 112\left(-\tfrac{1}{2}\right)$$

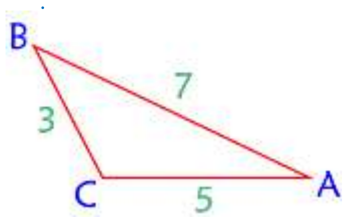
$$x^2 = 113 + 56$$

$$x^2 = 169$$

$$x = \pm 13 \quad \boxed{x = 13}$$

FINDING AN ANGLE

4)



$$5^2 = 3^2 + 7^2 - 2(3)(7)\cos B$$

$$\frac{25 - 9 - 49}{-2(3)(7)} = \cos B$$

$$B = \cos^{-1}\left(\frac{-33}{-42}\right) \quad B \approx 38.2^\circ$$

$$3^2 = 5^2 + 7^2 - 2(5)(7)\cos A$$

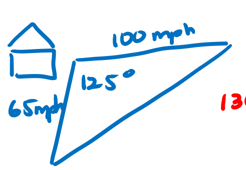
$$\frac{9 - 25 - 49}{-2(5)(7)} = \cos A$$

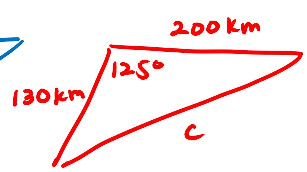
$$A = \cos^{-1}\left(\frac{65}{70}\right)$$

$$A \approx 21.8$$

$$\boxed{c = 120^\circ}$$

5) Two trains leave a station on different tracks. The tracks make an angle of 125° with the station as the vertex. The first train travels at an average speed of 100 km per hour and the second at an average of 65 km per hour. How far apart are the trains after 2 hours?





$$c^2 = 130^2 + 200^2 - 2(130)(200)\cos 125^\circ$$

$$c^2 = 86725.97$$

$$c = \pm 294.5$$

$$\boxed{294.5\text{ km}}$$