(PART 2) THE LAW OF SINES AND LAW OF COSINES

OBJECTIVES: 1) Use the law of sines and law of cosines in application problems.
2) Determine if there are one, two, or no triangles to a given problem.

THE AMBIGUOUS CASE: SSA

If you are given two angles and one side (ASA or AAS) the Law of Sines will provide you with ONE solution for a missing side. However, the Law of Sines has a problem dealing with SSA!

The Law of Sines could provide you with no solution, one solution, or possibly two!

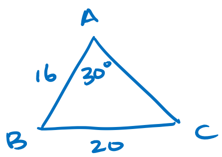
RECALL:

The sine function has a range of $[-1,1]$.

The $\sin^{-1}$ function has a domain of $[-1,1]$ and range of $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

If $\sin\theta$ is positive, θ can lie in the first or second quadrant.

1) In ΔABC, $a = 20$, $c = 16$, $\angle A = 30^\circ$. Solve the triangle. ONE TRIANGLE!



$\frac{\sin 30^\circ}{20} = \frac{\sin C}{16}$ $\qquad \sin C = \frac{16\cdot\frac{1}{2}}{20}$ $\qquad C = \sin^{-1}\left(\frac{2}{5}\right)$

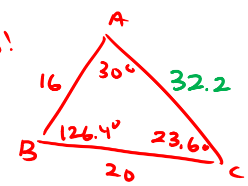
$\frac{16\sin 30^\circ}{20} = \sin C$ $\qquad \sin C = \frac{8}{20} = \frac{2}{5}$ $\qquad C \approx 23.58^\circ$ OR 156.42°

Doesn't work!

ONLY ONE Δ!

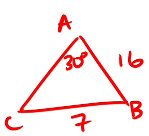
$\frac{b}{\sin 126.4^\circ} = \frac{20}{\sin 30^\circ}$

$b = \frac{20\sin 126.4^\circ}{\sin 30^\circ}$



$b = 32.2$

2) In ΔABC, $a = 7$, $c = 16$, $\angle A = 30^\circ$. Solve the triangle.



$\frac{\sin 30^\circ}{7} = \frac{\sin C}{16}$

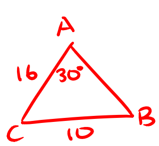
$\sin C = \frac{16\sin 30^\circ}{7}$

$C = \sin^{-1}\left(\frac{8}{7}\right)$ not in Domain of $\sin^{-1}(x)$!

There is NO Δ that exists!

THE AMBIGUOUS CASE!

3) In ΔABC, $a = 10$, $b = 16$, and $\angle A = 30^\circ$ ft. Solve the triangle. Two TRIANGLES!



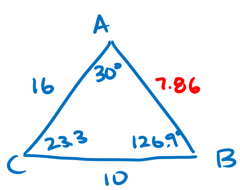
$\frac{\sin 30^\circ}{10} = \frac{\sin B}{16}$

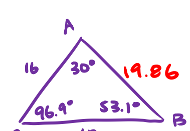
$\sin B = \frac{16\sin 30^\circ}{10}$

$B = \sin^{-1}(.8)$

BOTH work

$B \approx 53.13$ OR $B \approx 126.87$





$\frac{c}{\sin C} = \frac{10}{\sin 30^\circ}$

$c = \frac{10\sin C}{\sin 30^\circ}$

$c = \frac{10\sin 23.3}{\sin 30^\circ}$

$\boxed{c = 7.86}$

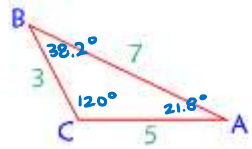
$\frac{c}{\sin C} = \frac{10}{\sin 30^\circ}$

$c = \frac{10\sin C}{\sin 30^\circ}$

$c = \frac{10\sin 96.9^\circ}{\sin 30^\circ}$

$\boxed{c = 19.86}$

4) Solve the triangle.



Use L.O.C. to find ∠C

$7^2 = 3^2 + 5^2 - 2(3)(5)\cos C$

$\frac{49-9-25}{-2(15)} = \cos C$

$C = \cos^{-1}\left(\frac{15}{-30}\right)$

$C = 120°$

$B \approx 38.21°$

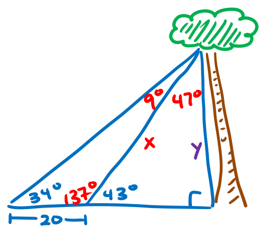
Use L.O.S. to find other ∠s

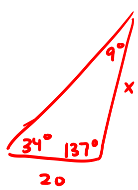
$\frac{\sin 120°}{7} = \frac{\sin A}{3}$ $\quad \sin A = \frac{3\sin 120°}{7}$ $\quad A = \sin^{-1}\left(\frac{3\sqrt{3}}{14}\right)$

$A \approx 21.79°$

APPLICATIONS OF LAW OF SINES/COSINES

5) You are an avid bird watcher! You spot the highly coveted yellow-headed blackbird in a tree north of where you are standing. You also have an excellent eye for angles, and you know from your current spot, the bird's nest is at an angle of elevation of 34°. You walk 20 feet closer to the bird's nest, to get a better look. Now, the angle of elevation is 43°. How far are you from the bird? How high is the bird's nest?





$\frac{20}{\sin 9°} = \frac{x}{\sin 34°}$

$\frac{20\sin 34°}{\sin 9°} = x$

$\boxed{x \approx 71.49 \text{ ft}}$

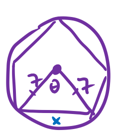
store

$\sin 43° = \frac{y}{x}$

$y = x\sin 43°$

$\boxed{y \approx 48.76 \text{ ft}}$

6) Find the perimeter and area of a regular pentagon inscribed in a circle of radius 7 (exact and approximate).



$\theta = \frac{360}{5} = 72°$

$P = 5x$

$x = \sqrt{7^2 + 7^2 - 2(7\cdot 7)\cos 72°}$

$x \approx 8.23$ (STO x!)

$\boxed{P \approx 41.145}$

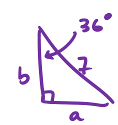
$A_\Delta = \frac{1}{2}ab\sin\theta$

$A_\Delta = \frac{1}{2}(7\cdot 7)\sin 72°$

$A_\Delta \approx 23.3$

$A_{⬠} = 5(A_\Delta) \approx \boxed{116.5}$

OR!



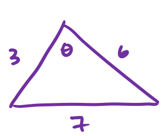
$\sin 36° = \frac{a}{7}$ $\quad \cos 36° = \frac{b}{7}$

$a = 7\sin 36°$ $\quad b = 7\cos 36°$

$A_\Delta = \frac{1}{2}(2a)b = \frac{1}{2}\cdot 2\cdot 7\sin 36\cdot 7\cos 36$

$= 49\sin 36\cos 36 \approx 23.3$

7) Find the area of a triangle with side lengths 3, 6, and 7.



$7^2 = 3^2 + 6^2 - 2(3)(6)\cos\theta$

$\frac{49-9-36}{-36} = \cos\theta$

$-\frac{1}{9} = \cos\theta$

$\cos^{-1}\left(-\frac{1}{9}\right) = \theta$

$\theta \approx 96.38°$

STO θ!

$A_\Delta = \frac{1}{2}ab\sin\theta$

$A_\Delta = \frac{1}{2}(3\cdot 6)\sin\theta$

$\boxed{A_\Delta \approx 8.94}$