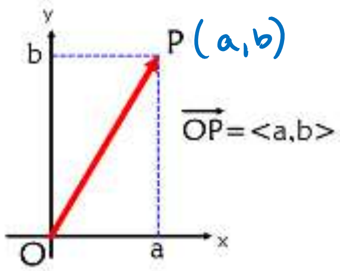
VECTORS: AN ALGEBRAIC APPROACH

OBJECTIVES: 1) Draw a vector in standard position and calculate the magnitude (or norm) of the vector.
2) Perform operations on vectors and find a unit vector.
3) Use vectors in navigation applications.

STANDARD POSITION

The **position vector** or the **standard vector** starts at the origin and uses <#,#> notation.



a is the horizontal component of $\overrightarrow{OP}$
b is the vertical component of $\overrightarrow{OP}$

$$\left|\overrightarrow{OP}\right|^2 = a^2 + b^2$$

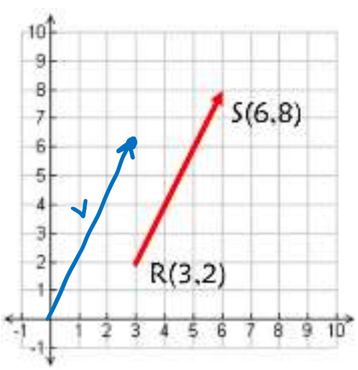
$$\left|\overrightarrow{OP}\right| = \sqrt{a^2+b^2}$$

LENGTH OF A VECTOR

If $\boldsymbol{v} = \langle v_1, v_2 \rangle$, then

$$|\boldsymbol{v}| = \sqrt{v_1^{\,2} + v_2^{\,2}}$$

1) Find the position vector of $\overrightarrow{RS}$ and label it **v**. Then find the length of **v**.



R(3,2) S(6,8)

$\overrightarrow{RS} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$\overrightarrow{RS} = \langle 6-3, 8-2 \rangle$

$\overrightarrow{RS} = \langle 3, 6 \rangle = v$

$|v| = \left|\overrightarrow{RS}\right| = \sqrt{3^2+6^2} = \sqrt{45} = \boxed{3\sqrt{5}}$

VECTOR ADDITION:

If $\boldsymbol{u} = \langle u_1, u_2 \rangle$ and $\boldsymbol{v} = \langle v_1, v_2 \rangle$, then $\boldsymbol{u} + \boldsymbol{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$

2) If $\mathbf{w} = \langle -2, 3 \rangle$ and $\mathbf{m} = \langle 5, 12 \rangle$, find $2\mathbf{w} - 3\mathbf{m}$ in both forms. Then find $|2\mathbf{w} - 3\mathbf{m}|$.

$2w - 3m = 2\langle -2,3 \rangle - 3\langle 5,12 \rangle$

$= \langle -4, 6 \rangle + \langle -15, -36 \rangle$

$= \langle -4 + -15, 6 + -36 \rangle$

$= \boxed{\langle -19, -30 \rangle}$ $\quad 2w - 3m = \boxed{-19i - 30j}$

$|2w - 3m| = \sqrt{(-19)^2 + (-30)^2} =$

UNIT VECTORS

$\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$

Use **i** and **j** to represent horizontal and vertical components:

$$\langle x, y \rangle = x\mathbf{i} + y\mathbf{j}$$

3) Find a unit vector **u** in the same direction as **m.**

$m = \langle 5, 12 \rangle \qquad |m| = \sqrt{5^2 + 12^2} = 13$

$u = \frac{1}{|m|} m = \frac{1}{13}\langle 5, 12 \rangle$

$= \boxed{\left\langle \frac{5}{13}, \frac{12}{13} \right\rangle}$

UNIT VECTOR WITH SAME DIRECTION AS V

If **v** is a nonzero vector, then

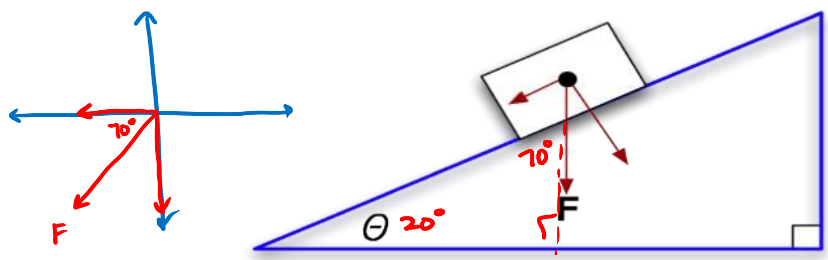
$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

COMPONENTS OF VECTORS

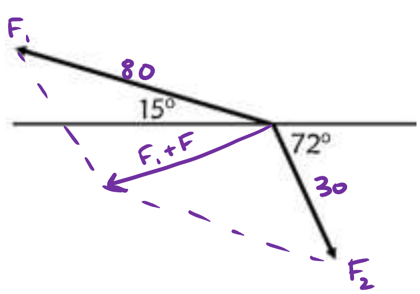
4) If an 18lb block rests on an inclined plane with a 20° angle of elevation, determine the components of the force perpendicular and parallel to the plane.

Parallel: $-18\cos 70°$

Perpendicular: $-18\sin 70°$



5) A force, F_1, of 80 pounds acts at an angle of 15° above the horizontal. Pulling in an opposing direction is force F_2 of 30 pounds acting at an angle of 72° below the horizontal. Find the horizontal and vertical components of the resultant force.



$F_{1x} = -80\cos 15°\, i \qquad F_{2x} = 30\cos 72°\, i$

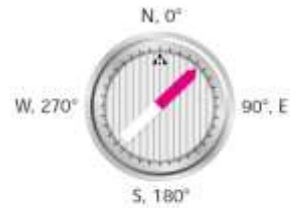
$F_{1y} = 80\sin 15°\, j \qquad F_{2y} = -30\sin 72°\, j$

$F_1 + F_2 = (-80\cos 15° + 30\cos 72°)\, i + (80\sin 15° - 30\sin 72°)\, j$

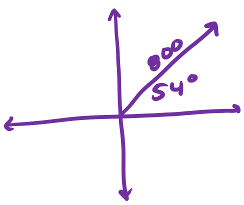
$\boxed{F_1 + F_2 = -68.0\, i + -7.83\, j}$

USING VECTORS IN NAVIGATION

HEADING: Clockwise from due north
AIR SPEED: Plane's speed (speedometer), plane alone
GROUND SPEED: Result of air and wind vectors (direction; also clockwise from N)
COURSE: Result of air and wind vectors (direction; also clockwise from N)
DRIFT ANGLE: Angle from heading to course (not from due N).



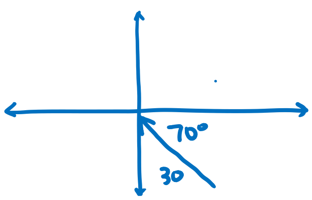
A plane has a heading of 36° with an airspeed of 800 km/hr.



$P_x = 800\cos 54°\, i$

$P_y = 800\sin 54°\, j$

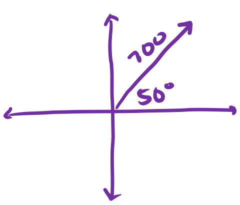
The wind is blowing from 160° at 30 km/hr.



$W_x = -30\cos 70\, i$

$W_y = 30\sin 70\, j$

6) A plane has a heading of 40° with an airspeed of 700 km/hr. The wind is blowing from 300° at 60 km/hr. Find the groundspeed, course and drift angle.



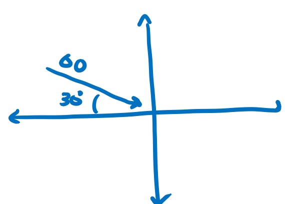
$P_x = 700\cos 50°\, i$

$P_x = 449.95\, i$

$P_y = 700\sin 50°\, j$

$P_y = 536.23\, j$

$P: 449.95i + 536.23j$



$W_x = 60\cos 30°\, i$ $W_y = -60\sin 30°\, j$

$W_x = 51.96°\, i$ $W_y = -30\, j$

$W: 51.96i - 30j$

$P+W = 501.91i + 506.23j$

$\theta = \tan^{-1}\left(\frac{506.23}{501.91}\right)$

$\theta = 45°$

$\boxed{\text{Course: } 44.75° \quad \text{Drift: } 4.75°}$

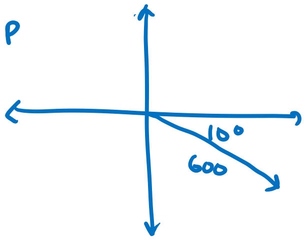
$|P+W| = \text{Ground speed}$

$= \sqrt{501.9^2 + 506.23^2}$

$= \boxed{712.86 \text{mph}}$

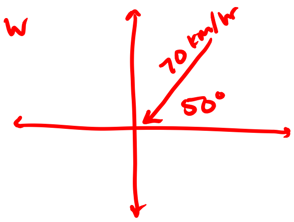
7) A plane is flying with an airspeed of 600 km/hr and heading 100°. The wind is 70 km/hr from 40°. Find the ground speed, drift angle and course.

P



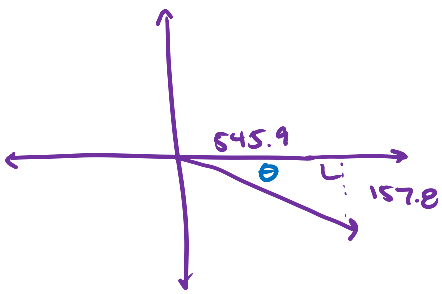
$P = 600\cos 10^\circ i - 600\sin 10^\circ j$

W



$W = -70\cos 50^\circ i - 70\sin 50^\circ j$

$P + W = 545.9i - 157.8j$



Groundspeed $= |P+W| = \sqrt{545.9^2 + 157.8^2} = \boxed{568.2 \text{ km/hr}}$

$\tan\theta = \dfrac{157.8}{545.9}$

$\theta = 16.1^\circ$

Course: $90 + 16.1^\circ = \boxed{106.1^\circ}$

Drift $\angle = |\text{Heading} - \text{Course}| = |100 - 106.1| = \boxed{6.1^\circ}$