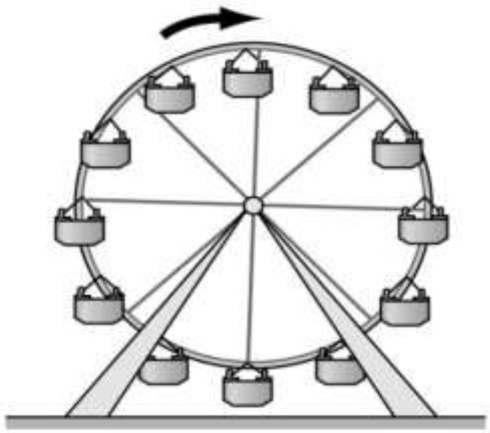
10.5
Notes
(Part 1)

PARAMETRIC EQUATIONS

OBJECTIVES: 1) Sketch curves that are represented by sets of parametric equations.
2) Eliminate the parameter to rewrite parametric equations as singular rectangular equations.

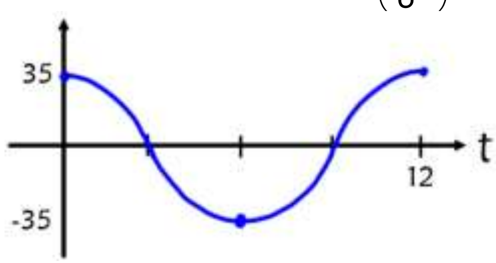
What is a **parametric equation**? A system of equations with more than one dependent variable. Often, parametric equations are used to represent the position of a moving point.

Imagine you are riding a Ferris wheel that has a 35' radius and whose lowest point is 5' off of the ground. It completes one clockwise rotation every 12 seconds. Your x and y position both depend on time. We call time the **parameter**.

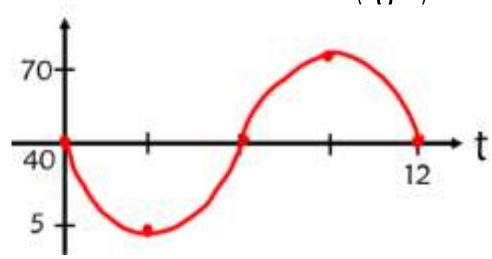


t	x	y
0	35	40
3	0	5
6	-35	40
9	0	75
12	35	40

x position: $x(t) = 35\cos\left(\frac{\pi}{6}t\right)$



y position: $y(t) = -35\sin\left(\frac{\pi}{6}t\right) + 40$



So $x(t) = 35\cos\left(\frac{\pi}{6}t\right)$ and $y(t) = -35\sin\left(\frac{\pi}{6}t\right) + 40$ are **parametric equations** that describe your position on the Ferris wheel at any time t.

PARAMETRIC EQUATIONS

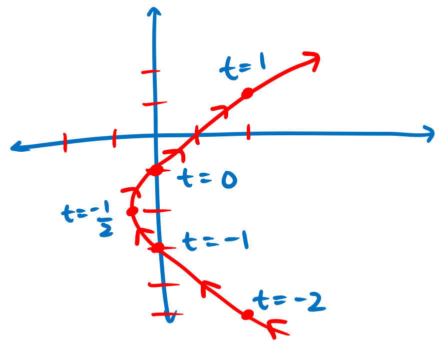
- If f and g are continuous functions of t on an interval I, then the set of ordered pairs (x, y) such that $x = f(t)$ and $y = g(t)$ is a **plane curve**.
- The equations $x = f(t)$ and $y = g(t)$ are **parametric equations** for the curve.
- The variable t is the **parameter.**
- Parametric equations have a definite direction of motion, called the **orientation** of the curve.

GRAPHING USING A TABLE

1) Sketch the parametric curve for the following set of parametric equations: $x = t^2 + t$ and $y = 2t - 1$

Select values of t, plug them into the parametric equations and plot the points:

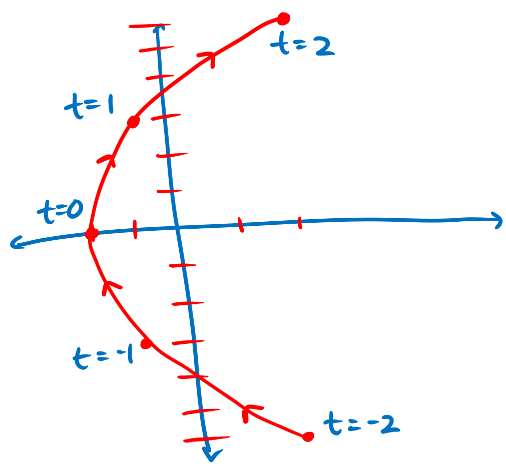
t	x	y
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-\frac{1}{4}$	-2
0	0	-1
1	2	1



2) Sketch the parametric curve for the following set of parametric equations:

$x = t^2 - 2$ $\quad y = 3t$ $\quad -2 \le t \le 2$

t	x	y
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3
2	2	6



GRAPHING BY ELIMINATING THE PARAMETER

3) Sketch the parametric curve for the following set of parametric equations:

$x = t^2 - 2$ $\quad y = 3t$ $\quad -2 \le t \le 2$

$x+2 = t^2$

$\pm\sqrt{x+2} = t$

So, $\boxed{y = \pm 3\sqrt{x+2}}$

OR

$\frac{y}{3} = t$

So, $x = \left(\frac{y}{3}\right)^2 - 2$

$\boxed{x = \frac{y^2}{9} - 2}$

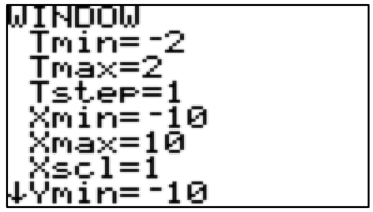
Then graph w/in the interval $-2 \le t \le 2$.

Eliminating the parameter is not always easy, and in some cases, it's not even possible.
In those cases, you would need to set up a table to graph.

GRAPHING ON YOUR CALCULATOR

4) Sketch the parametric curve for the following set of parametric equations:

$x = t^2 - 2 \qquad y = 3t \qquad -2 \le t \le 2$



5) Sketch the parametric curve for the set of parametric equations. Clearly indicate the direction of motion.

$x = 5\cos t \qquad y = 2\sin t \qquad 0 \le t \le 2\pi$

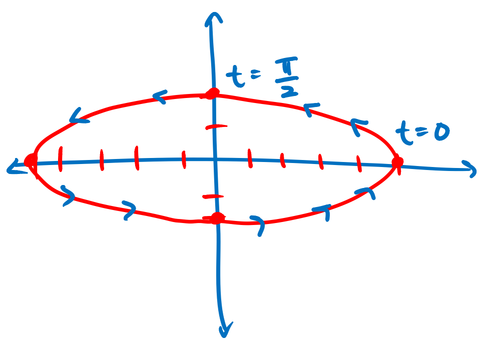
$\frac{x}{5} = \cos t \qquad \frac{y}{2} = \sin t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

Ellipse!



Direction of motion:

@ $t=0$ $\quad x = 5\cos 0$ & $y = 2\sin 0$

$x = 5 \quad y = 0$

@ $t = \frac{\pi}{2}$

$x = 5\cos\frac{\pi}{2}$ & $y = 2\sin\frac{\pi}{2}$

$x = 0 \quad y = 2$

∴ Counter clockwise

6) Sketch the parametric curve for the set of parametric equations. Clearly indicate the direction of motion.

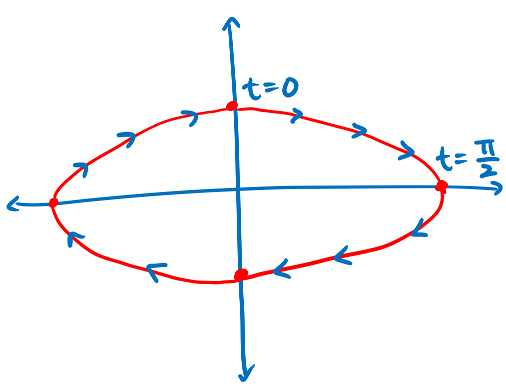
$x = 6\sin t \qquad y = 3\cos t \qquad 0 \le t \le 2\pi$

$\frac{x}{6} = \sin t \qquad \frac{y}{3} = \cos t$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{36} + \frac{y^2}{9} = 1$



@ $t=0$

$x = 6\sin 0 \quad y = 3\cos 0$

$x = 0 \quad y = 3$

@ $t = \frac{\pi}{2}$

$x = 6\sin\frac{\pi}{2} \quad y = 3\cos\frac{\pi}{2}$

$x = 6 \quad y = 0$

Orientation: clockwise