SYSTEMS OF LINEAR EQUATIONS

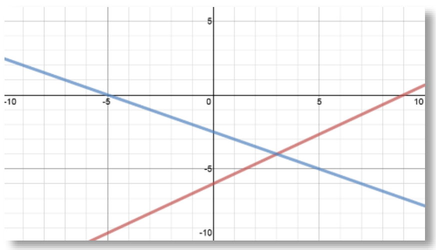
OBJECTIVES: 1) Use substitution and elimination to solve systems of equations in two variables.
2) Solve mixture problems and find the equation of a parabola through given points.

A SYSTEM OF EQUATIONS:

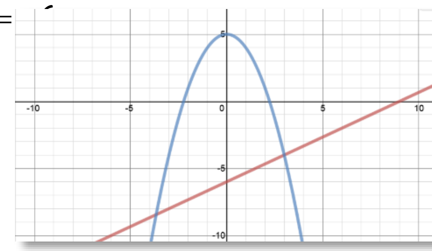
Definition: A system of equations is a **collection of two or more equations with a same set of unknowns**. In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

The equations in the system can be linear or non-linear. We focus on linear equations in 11.1.

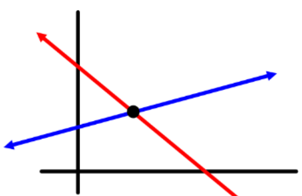
$$\begin{cases} 2x - 3y = 18 \\ x + 2y = -5 \end{cases}$$



$$\begin{cases} -2x + 3y = \\ x^2 + y = 5 \end{cases}$$

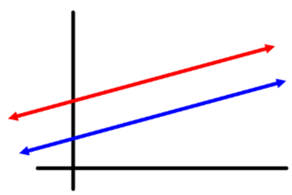


CONSISTENT



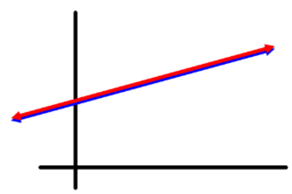
ONE SOLUTION

INCONSISTENT



NO SOLUTION

CONSISTENT



INFINITELY MANY SOLUTIONS

SOLVING A LINEAR SYSTEM BY SUBSTITUTION OR ELIIMINATION

Solve the following systems algebraically.

1) $\begin{cases} 3x - 6y = 12 \\ x - 2y = 6 \end{cases}$

$x = 2y + 6$

$3(2y+6) - 6y = 12$

$6y + 18 - 6y = 12$

$18 = 12$

No solution (Inconsistent)

2) $\begin{cases} \dfrac{4}{x} - \dfrac{3}{y} = 11 \\ \dfrac{5}{x} - \dfrac{6}{y} = 9 \end{cases}$

let $a = \frac{1}{x}$ $b = \frac{1}{y}$

$\boxed{(x, y)\ \left(\frac{3}{13}, \frac{9}{19}\right)}$

$-2(4a - 3b = 11)$

$5a - 6b = 9$

$-8a + 6b = -22$

$-3a = -13$

$a = \frac{13}{3}$

$-5(4a - 3b = 11)$

$4(5a - 6b = 9)$

$-20a + 15b = -55$

$20a - 24b = 36$

$-9b = -19$

$b = \frac{19}{9}$

Now take reciprocal!

APPLICATION: FINDING A QUADRATIC EQUATION

3) Determine the constants b and c so that the parabola $y = x^2 + bx + c$ passes through the points $(-2, 1)$ and $(2, 3)$.

plug in $(-2,1)$: $1 = (-2)^2 + (-2)b + c$ $\quad 1 = 4 - 2b + c$ $\quad -2b + c = -3$

plug in $(2,3)$: $3 = (2)^2 + 2b + c$ $\quad 3 = 4 + 2b + c$ $\quad 2b + c = -1$

$2c = -4$ $\quad -2b - 2 = -3$

$c = -2$ $\quad -2b = -1$

$b = \frac{1}{2}$

$$\boxed{y = x^2 + \frac{1}{2}x - 2}$$

APPLICATION: MIXTURE PROBLEMS

4) A pharmacist mixed some of a 30% iodine solution with some of a 40% iodine solution. How much of each solution would be needed to produce 200 milliliters of a 36% iodine solution?

x = # of mL of .30% iodine

y = # of mL of 40% iodine

$x + y = 200$ $\Rightarrow$ $x + y = 200$ $\quad -30x - 30y = -6000$

$.30x + .40y = .36(200)$ $\quad 30x + 40y = 7200$ $\quad 30x + 40y = 7200$

$10y = 1200$

$y = 120$

$x = 80$

$\boxed{\text{80 mL of 30\% and 120mL of 40\%}}$

YOU TRY

5) Calvin mixes candy that sells for \$2.00 per pound with candy that costs \$3.60 per pound to make 50 pounds of candy selling for \$2.16 per pound. How many pounds of each kind of candy did he use in the mix?

x = # lbs of \$2.00 candy

y = # lbs of \$3.60 candy

$x + y = 50$ $\Rightarrow$ $x + y = 50$ $\quad -20x - 20y = -1000$

$2x + 3.6y = 50(2.16)$ $\quad 20x + 36y = 1080$ $\quad 20x + 36y = 1080$

$16y = 80$

$y = 5$

$x = 45$

He mixes 45 lbs of \$2.00 candy and 5 lbs of \$3.00 candy