11.2 and 11.3
Notes
Day 2

GAUSSIAN ELIMINATION AND MATRICES

OBJECTIVES: 1) Use Gaussian elimination to solve a system of equations.
2) Add, subtract, and multiply matrices.

GAUSSIAN ELIMINATION

Solve the following systems of equations:

TRIANGULAR FORM

1) $\begin{cases} 5x + 4y - z = 0 \\ 10y - 3z = 11 \\ z = 3 \end{cases}$

$z = 3$ $\quad 10y - 3(3) = 11$

$10y = 20$

$y = 2$

Back substitution

$5x + 4(2) - 3 = 0$

$5x + 5 = 0$

$x = -1$

$(-1, 2, 3)$

2) $\begin{cases} -3x + 2y - 6z = 6 \\ 5x + 7y - 5z = 6 \\ x + 4y - 2z = 8 \end{cases}$

See Gaussian elimination method in the video!

Using inverse matrices:

Let $A = \begin{bmatrix} -3 & 2 & -6 \\ 5 & 7 & -5 \\ 1 & 4 & -2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 6 \\ 6 \\ 8 \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

So...

$AX = B$

$A^{-1} \cdot AX = B \cdot A^{-1}$ (switch order!)

I

$X = A^{-1}B$

$x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

INVERSE MATRICES

3) $\begin{cases} x + 2y - 7z = -4 \\ 2x + y + z = 13 \\ 3x + 9y - 36z = -33 \end{cases}$

$A = \begin{bmatrix} 1 & 2 & -7 \\ 2 & 1 & 1 \\ 3 & 9 & -36 \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\quad B = \begin{bmatrix} -4 \\ 13 \\ -33 \end{bmatrix}$

$X = A^{-1}B$

This gives us an error message!
So use rref!

ROW REDUCED ECHELON FORM

4) $\begin{cases} x + 2y - 7z = -4 \\ 2x + y + z = 13 \\ 3x + 9y - 36z = -33 \end{cases}$

let $A = \begin{bmatrix} 1 & 2 & -7 & -4 \\ 2 & 1 & 1 & 13 \\ 3 & 9 & -36 & -33 \end{bmatrix}$

$\text{rref}A = \begin{bmatrix} 1 & 0 & 3 & 10 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ indicates infinitely many solutions!

all possible solutions terms of z:

$(x(z), y(z), z)$

$(-3z + 10, 5z - 7, z)$

$1y - 5z = -7$ $\quad y = 5z - 7$

$1x + 3z = 10$ $\quad x = -3z + 10$

MATRICES

Give the dimension of each matrix:

3) $\begin{bmatrix} -12 & 1 \\ 2 & 3 \\ 1 & -5 \end{bmatrix}$ 3×2

4) $\begin{bmatrix} 1 & 4 & 3 \\ -2 & -8 & -6 \end{bmatrix}$ 2×3

5) $\begin{bmatrix} 11 \\ 4 \\ 21 \end{bmatrix}$ 3×1

ADDING/SUBTRACTING MATRICES AND SCALAR MULTIPLICATION

Note: The dimensions must be the same to be added or subtracted. A matrix can be multiplied by a scalar (a number) by multiplying each element of the matrix that scalar:

6) $\begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -9 \\ 4 & 10 \\ 7 & -4 \end{bmatrix}$

$\begin{bmatrix} 1 & -8 \\ 7 & 16 \\ 9 & -7 \end{bmatrix}$

7) $\begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 2 & -3 \end{bmatrix} - 2\begin{bmatrix} -1 & -9 \\ 4 & 10 \\ 7 & -4 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 18 \\ -8 & -20 \\ -14 & 8 \end{bmatrix}$

$\begin{bmatrix} 4 & 19 \\ -5 & -14 \\ -12 & 5 \end{bmatrix}$

8) $\begin{bmatrix} 2 & -10 & 15 \end{bmatrix} + \begin{bmatrix} 1 \\ -16 \\ -13 \end{bmatrix}$

Can't add!

MATRIX MULTIPLICATION

9) $\begin{bmatrix} 5 & -7 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -12 & 3 \\ 2 & 0 \end{bmatrix}$

$2\times 2 \cdot 2\times 2$

Can multiply → is a 2×2

$\begin{bmatrix} 5\cdot -12 + -7\cdot 2 & 5\cdot 3 + -7\cdot 0 \\ 1\cdot -12 + 4\cdot 2 & 1\cdot 3 + 4\cdot 0 \end{bmatrix}$

$\begin{bmatrix} -74 & 15 \\ -8 & 3 \end{bmatrix}$

Not the same!

10) $\begin{bmatrix} -12 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ 1 & 4 \end{bmatrix}$

$2\times 2 \cdot 2\times 2$

$\begin{bmatrix} -12\cdot 5 + 3\cdot 1 & -12\cdot -7 + 3\cdot 4 \\ 2\cdot 5 + 0\cdot 1 & 2\cdot -7 + 0\cdot 4 \end{bmatrix}$

$\begin{bmatrix} -57 & 96 \\ 10 & -14 \end{bmatrix}$

11) $\begin{bmatrix} 3 \\ 8 \\ -13 \end{bmatrix}\begin{bmatrix} 1 & 9 \end{bmatrix}$

$3\times 1 \cdot 1\times 2$

$\begin{bmatrix} 3\cdot 1 & 3\cdot 9 \\ 8\cdot 1 & 8\cdot 9 \\ -13\cdot 1 & -13\cdot 9 \end{bmatrix}$

$\begin{bmatrix} 3 & 27 \\ 8 & 72 \\ -13 & -117 \end{bmatrix}$

12) $\begin{bmatrix} 1 \\ -2 \\ 6 \\ 7 \end{bmatrix}\begin{bmatrix} 2 & -8 & 7 & -12 \end{bmatrix}$

$4\times 1 \cdot 1\times 4$

$\begin{bmatrix} 2 & -8 & 7 & -12 \\ -4 & 16 & -14 & 24 \\ 12 & -48 & 42 & -72 \\ 14 & -56 & 49 & -84 \end{bmatrix}$

13) $\begin{bmatrix} 1 & -3 \\ -2 & 5 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 10 & -11 \end{bmatrix}$

$3\times 2 \cdot 2\times 3$

$\begin{bmatrix} 1\cdot 2 + -3\cdot 10 & 1\cdot -4 + -3\cdot -11 \\ -2\cdot 2 + 5\cdot 10 & -2\cdot -4 + 5\cdot -11 \\ -3\cdot 2 + 7\cdot 10 & -3\cdot -4 + 7\cdot -11 \end{bmatrix}$

$\begin{bmatrix} -28 & 29 \\ 46 & -47 \\ 64 & -65 \end{bmatrix}$

14) $\begin{bmatrix} 2 & -4 \\ 10 & -11 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 5 \\ -3 & 7 \end{bmatrix}$

$2\times 2 \cdot 3\times 2$

Can't multiply!