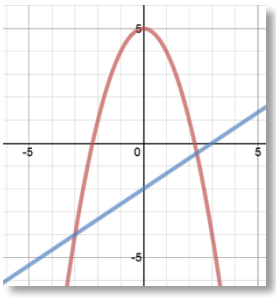
11.6
Notes

NONLINEAR SYSTEMS OF EQUATIONS

OBJECTIVES: 1) Algebraically solve a nonlinear system of equations.

GRAPHICALLY

Ex 1) $\begin{cases} -2x+3y=-6 \\ x^2+y=5 \end{cases}$



ALGEBRAICALLY

$y=-x^2+5$

$-2x+3(-x^2+5)=-6$

$-2x-3x^2+15=-6$

$-3x^2-2x+21=0$

$-(3x^2+2x-21)=0$

$-(3x-7)(x+3)=0$

$x=\frac{7}{3} \quad x=-3$

$\left(\frac{7}{3},\frac{-4}{9}\right) \quad (-3,-4)$

If $x=\frac{7}{3}$,

$y=-\left(\frac{7}{3}\right)^2+5$

$y=-\frac{49}{9}+\frac{45}{9}$

$y=-\frac{4}{9}$

If $x=-3$

$y=-(-3)^2+5$

$y=-4$

SOME MO' (BUT DIFFERENT) EXAMPLES

2) $\begin{cases} y=x^2 \\ x^2+y^2=1 \end{cases}$

Elimination

$-x^2+y=0$

$x^2+y^2=1$

$y^2+y=1$

$y^2+y-1=0$

$y=\frac{-1\pm\sqrt{1-4(-1)}}{2}$

$y=\frac{-1\pm\sqrt{5}}{2}$

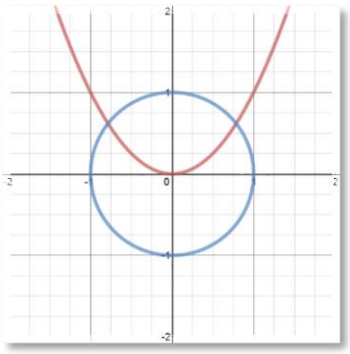
"y" can't be negative! throw out neg. #

If $y=\frac{-1+\sqrt{5}}{2}$

then $x=\pm\sqrt{\frac{-1+\sqrt{5}}{2}}$

$\left(\sqrt{\frac{-1+\sqrt{5}}{2}},\frac{-1+\sqrt{5}}{2}\right)$

$\left(-\sqrt{\frac{-1+\sqrt{5}}{2}},\frac{-1+\sqrt{5}}{2}\right)$



3) $\begin{cases} y=\sqrt{x} \\ (x+2)^2+y^2=1 \end{cases}$

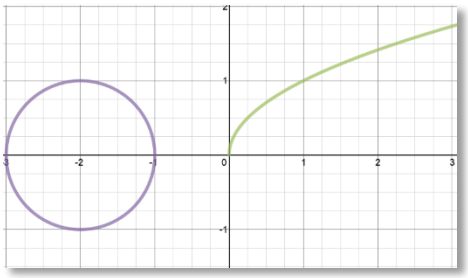
$(x+2)^2+x=1$

$x^2+4x+4+x=1$

$x^2+5x+3=0$

$x=\frac{-5\pm\sqrt{25-4(1)(3)}}{2}$

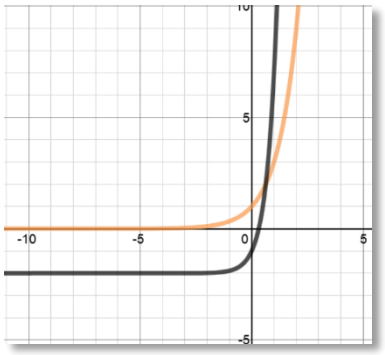
$x=\frac{-5\pm\sqrt{13}}{2}$ ← Both values are negative!



$y=\sqrt{x}$, means $x\geq 0$ so y is not real!

No solution to this system b/c they do not intersect!

4) $\begin{cases} y = 3^x \\ y = 3^{2x} - 2 \end{cases}$



$3^{2x} = (3^x)^2$

$y = y^2 - 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$

$y = -1$ ← not an output for $y = 3^x$!

If $y = 2$,

$2 = 3^x$

$\log_3 2 = x$

$(\log_3 2, 2)$

USE SUBSTITUTION!

5) $\begin{cases} \frac{2}{x^2} - \frac{3}{y^2} = -6 \\ \frac{3}{x^2} + \frac{4}{y^2} = 59 \end{cases}$

let $a = \frac{1}{x^2}$ $b = \frac{1}{y^2}$

$\begin{cases} 2a - 3b = -6 \\ 3a + 4b = 59 \end{cases}$

$6a - 9b = -18$

$-6a - 8b = -108$

$-17b = -126$

$b = 8$

$6a - 9(8) = -18$

$6a - 72 = -18$

$6a = 54$

$a = 9$

$a = 9$ $\quad a = \frac{1}{x^2}$ $\quad \frac{1}{x^2} = 9$ $\quad x^2 = \frac{1}{9}$ $\quad x = \pm\sqrt{\frac{1}{9}}$ $\quad x = \pm\frac{1}{3}$

$b = 8$ $\quad b = \frac{1}{y^2}$ $\quad \frac{1}{y^2} = 8$ $\quad y^2 = \frac{1}{8}$ $\quad y = \pm\sqrt{\frac{1}{8}}$ $\quad y = \pm\frac{1}{2\sqrt{2}}$

$\left(-\frac{1}{3}, \frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{3}, -\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{3}, \frac{1}{2\sqrt{2}}\right)\left(\frac{1}{3}, -\frac{1}{2\sqrt{2}}\right)$