THE FUNDAMENTAL THEOREM OF ALGEBRA

OBJECTIVES: 1) Express a polynomial in the factored form $a_n(x-r_1)(x-r_2)$.
2) Write the equation of a quadratic given specific roots.

THE FUNDAMENTAL THEOREM OF ALGEBRA:

Every polynomial equation of the form

$$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0 \qquad (n \geq 1.\ a_n \neq 0)$$

has at least one root within the complex number system. (This root may be a real number.)

THE LINEAR FACTORS THEOREM:

A **polynomial f(x)** with degree n can be expressed as the product of n linear factors.

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0 = a_n(x-r_n)(x-r_{n-1})\ldots(x-r_1)$

(The complex numbers r_k that appear in these factors are not necessarily all distinct,

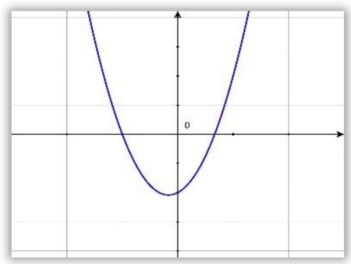
and some or all of the r_k may be real numbers.

1) Express $3x^2 + x - 2$ as $a_n(x-r_1)(x-r_2)$.

$(3x-2)(x+1)$

$3(x-\frac{2}{3})(x+1)$

$r_1 = \frac{2}{3} \quad r_2 = -1$

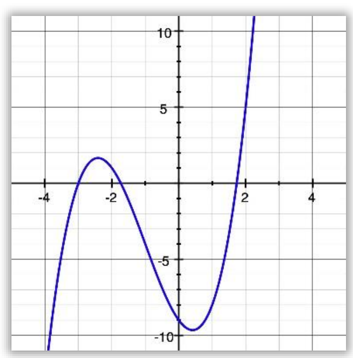


2) Express $x^3 + 3x^2 - 3x - 9$ as $a_n(x-r_1)(x-r_2)\ldots$.

$x^2(x+3) - 3(x+3)$

$(x^2-3)(x+3)$

$(x+\sqrt{3})(x-\sqrt{3})(x+3)$



3) Create **a** polynomial using the table below.

Root	Multiplicity
-1	2
3	1
0	2

$f(x) = x^2(x-3)(x+1)^2$

4) Find the EXACT quadratic polynomial with roots -1 and 2 and through (6,2).

$f(x) = a(x+1)(x-2)$

$y = a(x+1)(x-2)$

$2 = a(7)(4)$

$2 = 28a$

$a = \frac{1}{14}$

$\boxed{f(x) = \frac{1}{14}(x+1)(x-2)}$

APPLYING THE LINEAR FACTORS

$$x^2 + bx + c = 0$$

$$r_1 \bullet r_2 = c \qquad r_1 + r_2 = -b$$

Proof:

$x^2 + bx + c = (x - r_1)(x - r_2)$

$x^2 + bx + c = x^2 - r_1 x - r_2 x + r_1 r_2$

$(-r_1 - r_2) = b$ $\boxed{c = r_1 r_2}$

$-(r_1 + r_2) = b$

$\boxed{r_1 + r_2 = -b}$

5) Find a quadratic equation with roots $r_1 = 2 - 3i$ and $r_1 = 2 + 3i$.

$c = r_1 \cdot r_2$

$c = (2-3i)(2+3i)$

$4 - 9i^2$

$4 + 9$

$c = \boxed{13}$

$-b = r_1 + r_2$

$-b = (2-3i) + (2+3i)$

$-b = 4$

$\boxed{b = -4}$

$\boxed{x^2 - 4x + 13 = 0}$