13.6 and 13.7
Notes
Day 3

PARTIAL FRACTIONS

OBJECTIVES: 1) Rewrite a fractional expression as the sum of two simpler fractions.

PARTIAL FRACTIONS

Process of rewriting a function into simpler functions.

1) If $ax+b=7x-12$ for all x, then $a=7$ and $b=-12$
2) If $px^3+qx^2+rx+s=-4x^2+1$ for all x, then $p=0,\ q=-4,\ r=0,$ and $s=1$.

2 ASSUPMTIONS:
1) Proper fraction
2) Denominator is factorable

WHY? Calculus! It's easier to write a nasty function in terms of smaller functions when dealing with antiderivatives and other calculus topics

Rewrite the following as a sum or difference of partial fractions:

1) $\dfrac{5x-1}{x^2-2x-15}$

Method 1:

$\dfrac{5x-1}{(x-5)(x+3)} = \dfrac{A}{(x-5)} + \dfrac{B}{(x+3)}$

$5x-1 = A(x+3)+B(x-5)$

SYSTEM TO SOLVE
$\begin{cases} 5=A+B \\ -1=3A-5B \end{cases}$

$-15=-3A-3B$

$-16=-8B$

$B=2$

$A=3$

$\dfrac{3}{x-5}+\dfrac{2}{x+3}$

Method 2:

$5x-1=A(x+3)+B(x-5)$

$x=-3$

$5(-3)-1=A(0)+B(-8)$

$-16=-8B$

$B=2$

$x=5$

$5(5)-1=A(5+3)+B(0)$

$24=8A$

$A=3$

REPEATED LINEAR

2) $\dfrac{-x^2+2x+4}{x^3-4x^2+4x}$

$\dfrac{-x^2+2x+4}{x(x-2)^2} = \dfrac{A}{x}+\dfrac{B}{(x-2)}+\dfrac{C}{(x-2)^2}$

USE COMBO. OF METHOD 1 & 2

$-x^2+2x+4=A(x-2)^2+B(x-2)(x)+Cx$

let $x=0$

$4=A(-2)^2+B(0)+C(0)$

$4=4A$

$A=1$

Equate Coefficients:

$-1=A+B \quad (A=1)$

$-1=1+B$

$B=-2$

$2=-4A-2B+C$

$2=-4+4+C$

$C=2$

$\dfrac{1}{x}+\dfrac{-2}{(x-2)}+\dfrac{2}{(x-2)^2}$

IRREDUCIBLE QUADRATIC

3) $\dfrac{x^2+4x+1}{x^3-x^2+x-1} = \dfrac{x^2+4x+1}{x^2(x-1)+1(x-1)} = \dfrac{x^2+4x+1}{(x^2+1)(x-1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$

$x^2+4x+1 = A(x^2+1)+(Bx+C)(x-1)$

$x=1$

$1+4+1 = A(2)+(Bx+C)(0)$

$6=2A$

$\boxed{A=3}$

Equate Coefficients:

x^2:

$1 = A+B$

$1 = 3+B$

$\boxed{B=-2}$

constant:

$1 = A-C$

$1 = 3-C$

$\boxed{C=2}$

$\dfrac{3}{x-1} + \dfrac{-2x+2}{x^2+1}$

$\boxed{\dfrac{3}{x-1} - \dfrac{2(x-1)}{x^2+1}}$

YOU TRY!

4) $\dfrac{x-1}{x^2-x-6}$ You'll get a fraction, don't think it's wrong!

$\dfrac{x-1}{(x-3)(x+2)} = \dfrac{A}{(x-3)} + \dfrac{B}{(x+2)}$

$x-1 = A(x+2)+B(x-3)$

$x=-2$

$-3 = A(0)+B(-5)$

$\boxed{B=\frac{3}{5}}$

$x=3$

$2 = A(5)+B(0)$

$\boxed{A=\frac{2}{5}}$

$\dfrac{\frac{2}{5}}{(x+2)} + \dfrac{\frac{3}{5}}{(x-3)} \Rightarrow \boxed{\dfrac{2}{5(x+2)} + \dfrac{3}{5(x-3)}}$