14.1
Notes
Day 2

MATHEMATICAL INDUCTION (AGAIN!)

OBJECTIVES: 1) Use mathematical induction to prove a statement involving an inequality.

WRITING A PROOF BY INDUCTION

A proof by mathematical induction that a proposition P(n) is true for every positive integer n consists of two steps:

BASE CASE: Show that the proposition P(1) is true.

INDUCTIVE STEP: Assume that P(k) is true for an arbitrarily chosen positive integer k, and show that under that assumption, P(k+1) must be true.

From these two steps we conclude (by the principle of mathematical induction) that for all positive integers n, P(n) is true.

PROVING A STATEMENT THAT IS AN INEQUALITY

1) Use mathematical induction to show that the statement is true for all natural numbers:
$3^n \geq 1+2n$ for all $n \geq 1$

BASE CASE:

P(1): $3^1 \geq 1+2$

$3 \geq 3$ ✓

INDUCTIVE STEP:

Assume P(k) is true. Show that P(k+1) is also true.

If $3^k \geq 1+2k$ then $3^{k+1} \geq 1+2(k+1)$

$3^{k+1} \geq 2k+3$

Start with "if"

$3(3^k) \geq (1+2k)3$

$3^{k+1} \geq 3+6k$ ← "small" side, replace it w/ something even smaller

$3^{k+1} \geq 2k+3$

∴ P(n) is true for all natural numbers.

Side proof:

Show that: $3+6k \geq 2k+3$

$k \geq 0$

$4k \geq 0$

$3+6k \geq 2k+3$

2) Use mathematical induction to show that the statement is true for all natural numbers:

$(1.25)^n > n$ for all $n \geq 11$

Base Case: Show that P(11) is true.

P(11): $(1.25)^{11} > 11$

$11.64 > 11$ is true.

Inductive Step: Assume P(k) is true. Show that P(k+1) is true.

If $(1.25)^k > k$, then $1.25^{k+1} > k+1$

$1.25(1.25^k) > 1.25k$

$1.25^{k+1} > 1.25k$

Side proof: Show that $1.25k > k+1$

$k \geq 11$ ← start w/ a true statement

$.25k \geq .25(11)$

$.25k > 1$ since $.25(11) > 1$

now add k to both sides

$k + .25k > k+1$

$1.25k > k+1$

So, $1.25^{k+1} > k+1$

∴ P(n) is true.