14.1
Notes
Day 3

MATHEMATICAL INDUCTION (AGAIN! AGAIN!)

OBJECTIVES: 1) Use mathematical induction to prove a statement about factors.

WRITING A PROOF BY INDUCTION

A proof by mathematical induction that a proposition P(n) is true for every positive integer n consists of two steps:

BASE CASE: Show that the proposition P(1) is true.

INDUCTIVE STEP: Assume that P(k) is true for an arbitrarily chosen positive integer k, and show that under that assumption, P(k+1) must be true.

From these two steps we conclude (by the principle of mathematical induction) that for all positive integers n, P(n) is true.

PROVING A STATEMENT ABOUT FACTORS

1) Use mathematical induction to show that the statement is true for all natural numbers:
$23^n - 1$ is divisible by 11

Base Step: P(1) is true.

For $n=1$, $23^1 - 1$ is divisible by 11 is true.

Inductive Step: Assume that P(k) is true. Show that P(k+1) is true.

If: $11w_1 = 23^k - 1$ then: $11w_2 = 23^{k+1} - 1$

$11w_1 = 23^k - 1$

$23(11w_1) = 23(23^k - 1)$ multiply by 23 to match exponents on r.h.s.

$253w_1 = 23^{k+1} - 23$
$-22 \qquad -22$ subtract 22 to match R.H.S.

$253w_1 - 22 = 23^{k+1} - 1$

$11(23w_1 - 2) = 23^{k+1} - 1$ $23w_1 - 2$ is just another whole number. Replace it.

$11w_2 = 23^{k+1} - 1$

Since P(k+1) is true, P(n) is true for all natural numbers.

2) Use mathematical induction to show that the statement is true for all natural numbers:

$2^{2n} - 1$ is divisible by 3

BASE STEP: P(1) is true.

For $n=1$, $2^{2(1)}-1 = 4-1 = 3$ is divisible by 3.

INDUCTIVE STEP: Assume P(k) is true. Show that P(k+1) is true.

If P(k) is true, then P(k+1) is true.

If: $3w_1 = 2^{2k}-1$ then: $3w_2 = 2^{2(k+1)}-1$

$3w_2 = 2^{2k+2}-1$

$3w_1 = 2^{2k}-1$

$2^2(3w_1) = 2^2(2^{2k}-1)$ mult. by 2^2 to match exponents on R.H.S.

$12w_1 = 2^{2k+2}-4$ subtract 3 to match R.H.S.

$-3 \qquad -3$

$12w_1 - 3 = 2^{2k+2}-1$ (R.H.S. is good!)

$3(4w_1 - 1) = 2^{2k+2}-1$ Factor L.H.S.

$4w_1 - 1$ is just a whole number. Replace it.

$3w_2 = 2^{2k+2}-1$

P(k+1) is true. $\therefore$ P(n) is true for all natural numbers.