14.2
Notes
Day 1

# Binomial Theorem (Part 1)

**Objectives**: 1) Use Pascal's Triangle to complete binomial expansion.

## Binomial Expansions

Find the product.

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

### Patterns in Expansion of $(A+B)^N$:

- There are n + 1 terms.
- The expansion begins with $a^n$ and ends with $b^n$
- The sum of the exponents in each term is n.
- The exponents of a decrease by 1 from term to term.
- The exponents of b increase by 1 from term to term.
- When n is even, the coefficients are symmetric about the middle term.
- When n is odd, the coefficients are symmetric about the two middle terms.

The triangle was studied by Blaise Pascal (born 1623), although it had been described centuries earlier by Chinese mathematician Yanghui (about 500 years earlier, in fact) and the Persian astronomer-poet Omar Khayyám. It is therefore known as the Yanghui triangle in China.

### Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

## ANOTHER WAY TO FIND COEFFICIENTS OF ANY TERM:

It would be inefficient to use Pascal's triangle for expansions much higher than 6, so another pattern is useful when doing an expansion of $(a+b)^n$.

The coefficient of any term can be generated as follows: From the ***previous*** term, multiply the coefficient by the exponent of '$a$' and then divide by the number of that previous term.

For example, let's compute the second and third coefficients of the expansion of $(a+b)^7$ to compute the coefficient of the second term, refer back to the first term $a^7$.

coefficient of second term $= \frac{1 \bullet 7}{1} = 7$ coefficient of third term $= \frac{7 \bullet 6}{2} = 21$

**Expand the binomial using Pascal's Triangle.**

1) $(x-2)^4$

$1\,(x)^4(-2)^0 + 4\,(x)^3(-2)^1 + 6\,(x)^2(-2)^2 + 4\,(x)^1(-2)^3 + 1\,(x)^0(-2)^4$

$\boxed{x^4 - 8x^3 + 24x^2 - 32x + 16}$

2) $(3x+2y)^6$

$1\,(3x)^6 + 6\,(3x)^5(2y)^1 + 15\,(3x)^4(2y)^2 + 20\,(3x)^3(2y)^3 + 15\,(3x)^2(2y)^4 + 6\,(3x)(2y)^5 + 1\,(2y)^6$

$729x^6 + 6(243x^5)(2y) + 15 \cdot 81x^4 \cdot 4y^2 + 20(27x^3)(8y^3) + 15(9x^2)(16y^4) + 6(3x)(32y^5) + 729/2^6)y^6$

$\boxed{729x^6 + 2916x^5y + 4860x^4y^2 + 4320x^3y^3 + 2160x^2y^4 + 576xy^5 + 46{,}656y^6}$

3) $(2x-5)^6$

$1\,(2x)^6 + 6\,(2x)^5(-5) + 15\,(2x)^4(-5)^2 + 20\,(2x)^3(-5)^3 + 15\,(2x)^2(-5)^4 + 6\,(2x)(-5)^5 + 1\,(-5)^6$

$\boxed{64x^6 - 960x^5 + 6000x^4 - 20{,}000x^3 + 37500x^2 - 37500x + 15625}$