14.2
Notes
Day 2

BINOMIAL THEOREM (PART 2)

OBJECTIVES: 1) Evaluate or simplify expressions containing factorials or binomial coefficients.
2) Use Binomial Theorem to complete binomial expansion

DEFINITION: THE FACTORIAL SYMBOL

$n! = 1 \bullet 2 \bullet 3 \cdots n$ where n is a natural number.

$0! = 1$

Ex: $\frac{7!}{5!} = \frac{7 \bullet 6 \bullet \cancel{5} \bullet \cancel{4} \bullet \cancel{3} \bullet \cancel{2} \bullet \cancel{1}}{\cancel{5} \bullet \cancel{4} \bullet \cancel{3} \bullet \cancel{2} \bullet \cancel{1}} = 7 \cdot 6 = 42$

1) $\frac{(n+2)!}{(n-1)!}$ $\frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$(n+2)(n+1)(n) = (n^2+3n+2)n$

$\boxed{= n^3+3n^2+2n}$

2) $\frac{(n+3)!}{(n+1)!}$ $\frac{(n+3)(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}} = \boxed{n^2+5n+6}$

DEFINITION: THE BINOMIAL COEFFICIENT $\binom{n}{k}$: "n choose k"

Let n and k be nonnegative integers, with $k \le n$: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

3) $\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{2 \cdot 1(\cancel{3!})} = \boxed{10}$

4) $\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}\,(2)!} = \frac{20}{2 \cdot 1} = \boxed{10}$

Binomial coefficients are named so because they are the coefficients of the expansion of $(a+b)^n$. The coefficients of $(a+b)^n$ are the n+1 numbers $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n}$.

THE BINOMIAL THEOREM:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Expand the binomial (only the first 4 terms) using the binomial theorem.

1) $(x-2)^{31}$

$\binom{31}{0}x^{31} + \binom{31}{1}x^{30}(-2)^1 + \binom{31}{2}x^{29}\cdot(-2)^2 + \binom{31}{3}x^{28}(-2)^3$

$1x^{31} + 31x^{30}(-2) + 465x^{29}\cdot 4 + (4495)x^{28}\cdot(-8)$

$\boxed{x^{31} - 62x^{30} + 1860x^{29} - 35960x^{28}}$

2) $\left(\sqrt{x}+\frac{i}{x}\right)^{46}$

$\binom{46}{0}(\sqrt{x})^{46} + \binom{46}{1}\sqrt{x}^{45}\left(\frac{i}{x}\right) + \binom{46}{2}\sqrt{x}^{44}\left(\frac{i}{x}\right)^2 + \binom{46}{3}\sqrt{x}^{43}\left(\frac{i}{x}\right)^3$

$1x^{23} + 46x^{45/2}\cdot\frac{i}{x} + 1035\,x^{22}\left(\frac{-1}{x^2}\right) + 15180\,x^{43/2}\cdot\frac{i^3}{x^3}$

$x^{23} + 46i\cdot x^{43/2} - 1035\,x^{20} + 15180\,x^{37/2}(-i)$

$\boxed{x^{23} + 46i\cdot x^{43/2} - 1035\,x^{20} - 15180i x^{37/2}}$

3) $\left(\sqrt{2}-\sqrt{y}\right)^{24}$

$\binom{24}{0}(\sqrt{2})^{24} + \binom{24}{1}(\sqrt{2})^{23}(-\sqrt{y}) + \binom{24}{2}(\sqrt{2})^{22}(-\sqrt{y})^2 + \binom{24}{3}(\sqrt{2})^{21}(-\sqrt{y})^3$

$1\cdot 2^{12} + 24\cdot 2^{23/2}(-\sqrt{y}) + 276\cdot 2^{11}\cdot y + 2024\cdot 2^{21/2}(-y^{3/2})$

$4096 - 24\cdot 2^{23/2}(\sqrt{y}) + 276(2048)\cdot y - 2024\cdot 2^{21/2}y^{3/2}$

$\boxed{4096 - 69511.42502\cdot\sqrt{y} + 565248y - 2931065.088y^{3/2}}$