INTRO TO SEQUENCES AND SERIES

OBJECTIVES: 1) Find the first four terms of a sequence defined generally or recursively.
2) Use sigma notation to rewrite the sum of a sequence.

SEQUENCES

A numerical sequence is an ordered list of numbers. Ex 1) 1, 6, 10 Ex 2) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

A **term** is an individual entry in the sequence. Example 1 is a **finite sequence** since it only has a finite number of terms. Example 2 is an **infinite sequence** denoted with three dots read as "and so on".

A sequence is a function whose domain is the set of natural numbers. Instead of using f(x) notation, however, a sequence is listed using the a_n notation. The domain of an infinite sequences is the set of all positive integers, and the domain of a finite sequence is the set of the first n positive integers.

There are two common ways to define a sequence by specifying the general term.

GENERAL TERM, A_N

The first is to use a form that only depends on the number of the term, n. To find the first five terms when you know the general term, simply substitute the values 1, 2, 3, 4, and 5 into the general form for n and simplify.

1) Find the first 3 terms of the sequence if $a_n = \frac{n}{n+1}$.

$n=1$ $n=2$ $n=3$

$a_1 = \frac{1}{1+1}$ $a_2 = \frac{2}{2+1}$ $a_3 = \frac{3}{3+1}$

$a_1 = \frac{1}{2}$ $a_2 = \frac{2}{3}$ $a_3 = \frac{3}{4}$

RECURSIVE DEFINITION

The second way is to recursively define a sequence. A recursive definition uses the current and/or previous terms to define the next term. You can think of a_{n+1} being the next term, a_n being the current term, and a_{n-1} being the previous term.

2) Find the first 3 terms of the sequence if $a_1 = 5$ and $a_{n+1} = 2a_n - 1$. (You can read that last part as "the next term is one less than twice the current term")

$a_1 = 5$ $a_2 = 2(a_1) - 1$ $a_3 = 2(a_2) - 1$

$a_2 = 2(5) - 1$ $a_3 = 2(9) - 1$

$a_2 = 9$ $a_3 = 17$

GRAPHING A SEQUENCE

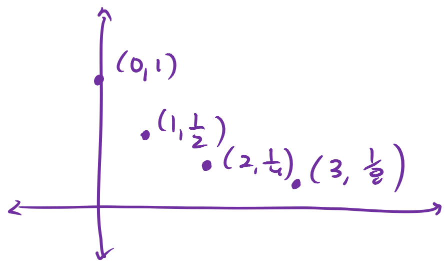
3) Graph the sequence (for n=0, 1, 2, and 3) defined by $x_n = \dfrac{1}{2^n}$ for $n \geq 0$.

$x_0 = \frac{1}{2^0} = 1$

$x_1 = \frac{1}{2^1} = \frac{1}{2}$

$x_2 = \frac{1}{2^2} = \frac{1}{4}$

$x_3 = \frac{1}{2^3} = \frac{1}{8}$



SIGMA NOTATION

To indicate the sum of the terms of a sequence, we use the capital Greek letter sigma, and so the notation is sometimes called Sigma Notation instead of Summation Notation.

$$\sum_{k=1}^{n} a_k = a_1 + a + a_3 + \ldots + a_n \quad \text{or} \quad \sum_{j=1}^{n} a_j = a_1 + a + a_3 + \ldots + a_n \quad \text{Ex: } \sum_{j=1}^{4} j^2 =$$

The "k" (or "j") is called the index of summation. k=1 is the lower limit of the summation and k=n (although the k is only written once) is the upper limit of the summation. What the summation notation means is to evaluate the argument of the summation for every value of the index between the lower limit and upper limit (inclusively) and then add the results together.

Express each of the following sums without sigma notation.

4) $\sum_{j=1}^{3} (3k-2)^2 = (3(1)-2)^2 + (3(2)-2)^2 + (3(3)-2)^2$

$= 1^2 + 4^2 + 7^2$

$= 1+16+49$

$\boxed{66}$

5) $\sum_{i=1}^{4} ix^{i-1} = 1 \cdot x^{1-1} + 2x^{2-1} + 3x^{3-1} + 4x^{4-1}$

$x^0 + 2(x^1) + 3x^2 + 4x^3$

$\boxed{1 + 2x + 3x^2 + 4x^3}$

USING SIGMA NOTATION

Use sigma notation to rewrite each sum.

6) $\dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^{12}}{12!}$

$\boxed{\sum_{k=1}^{12} \frac{x^k}{k!}}$

7) $\dfrac{x}{2!} + \dfrac{x^2}{3!} + \dfrac{x^3}{4!} + \ldots + \dfrac{x^n}{(n+1)!}$

$\boxed{\sum_{k=1}^{n} \frac{x^k}{(k+1)!}}$