14.4
Notes

ARITHMETIC SEQUENCES AND SERIES

OBJECTIVES: 1) Find a specific term/common difference in an arithmetic sequence.
2) Find the partial sum of an arithmetic sequence.

ARITHMETIC SEQUENCE: (ARITHMETIC PROGRESSION)

Sequence in which to move from one term to the next, you add the same constant for each successive term. i.e., the same number is ADDED to each previous term

Examples: 2, 5, 8, 11, 14,... and 7, 3, –1, –5,... d = common difference

d = 3 d = -4

THE NTH TERM OF AN ARITHMETIC SEQUENCE

$$a_n = a_1 + (n-1)d$$

1) Find the 40th term of 7, 12, 17, ... $d=5$

$a_{40} = 7 + (40-1)(5)$ $a_{40} = 7+(39)5 = \boxed{202}$

2) Determine which term a specific number is in an arithmetic sequence.

a) 101 with $a_1 = 5$ and d = 3.

$a_n = 101 = 5 + (n-1)3$

$101 = 5 + 3n - 3$

$99 = 3n$ $n = 33$

101 is the 33rd term

b) -60 when $a_1 = 17$ and d = -11

$a_n = -60$

$-60 = 17 + (n-1)\cdot -11$

$-60 = 17 - 11n + 11$

$-88 = -11n$ $n = 8$

-60 is the 8th term

3) Determine the arithmetic sequence in which the second term is -2 and the eighth term is 40.

$a_2 = -2$ $a_8 = 40$

$a_8 = 40 = a_1 + (8-1)d$

$a_2 = -2 = a_1 + (2-1)d$ → System!

$\begin{cases} a_1 + 7d = 40 \\ a_1 + 1d = -2 \end{cases}$

$6d = 42$

$d = 7$ $a_1 = -9$

$\boxed{a_n = -9 + (n-1)\cdot 7}$

4) a = 3, d = 2; Find the 75th term.

$a = 3$ $d = 2$ $n = 75$

$a_{75} = 3 + 2(75-1)$

$= 3 + 2(74)$

$= 151$

b) Find the indicated term: $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, \ldots, a_{30}$

$d = \frac{2}{5}$

$a_{30} = \frac{2}{5} + \frac{2}{5}(30-1)$

$a_{30} = \frac{2}{5} + \frac{2}{5}(29)$

$\boxed{a_{30} = 12}$

5) Find the common difference in an arithmetic sequence with $a_{10} - a_{20} = 70$.

$a_{10} - a_{20} = 70$

$a_{10} = a_1 - (10-1)d$

$a_{20} = a_1 - (20-1)d$

$a_1 - 9d = a_{10}$

$a_1 - 19d = a_{20}$

$a_{10} - a_{20} = 70$

$(a_1 - 9d) - (a_1 - 19d) = 70$

$10d = 70$

$\boxed{d = 70}$

ARITHMETIC SERIES

A series is a sum of a sequence. We want to find the n^{th} partial sum or the sum of the first n terms of the sequence. We will denote the n^{th} partial sum as S_n.

Consider the arithmetic sequence: 1 + 2 + 3 + 4 + . . . + 97 + 98 + 99 + 100

How can we find the sum quickly? Add 1+100, 2+99, 3+98...etc
How many pairs of numbers are there? 50
How many terms? 100
What do the pairs add up to? 101

sum:
$50(101) = 5050$

$S_n = \frac{100}{2}(101)$

PARTIAL SUM OF ARITHMETIC SERIES

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = n\left(\frac{a_1 + a_n}{2}\right) \quad \text{or} \quad S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

6) Find the sum of the first 50 terms with first = –8 and 50th = 139.

$S_n = 50\left(\frac{-8+139}{2}\right)$

$S_n = \frac{50(131)}{2} = 3,275$

7) The 5th and 50th terms of an arithmetic sequence are 3 and 30 respectively. Find the sum of the first 10.

$a_5 = 3 \quad a_{50} = 30 \quad$ Find S_{10}.

$a_{50} = 30 = a_1 + (50-1)d \quad a_1 + 49d = 30$
$a_5 = 3 = a_1 + (5-1)d \quad a_1 + 4d = 3$

$45d = 27$
$d = \frac{27}{45} \quad a_1 = .6$

$a_{10} = .6 + (10-1)\frac{27}{45}$
$= .6 + 9\left(\frac{27}{45}\right)$
$= 6$

$S_{10} = 10\left(\frac{.6+6}{2}\right) = 33$

8) Find $\sum_{n=5}^{100}(2n-1)$

Find the sum of the 5th through the 100th terms.

$a_5 = 2(5)-1 = 9$

$a_{100} = 2(100)-1 = 199$

$S = 96\left(\frac{9+199}{2}\right) = 9984$

There are 96 terms between n=5 & n=100.