14.5
Notes

GEOMETRIC SEQUENCES AND SERIES

OBJECTIVES: 1) Find a specific term/common ratio in a geometric sequence.
2) Find the sum of finite or infinite geometric series.

GEOMETRIC SEQUENCE: (GEOMETRIC PROGRESSION)

Sequence in which to move from one term to the next, you multiply by the same value for each successive term. i.e., the same number is MULTIPLIED to each previous term. A geometric sequence is an exponential function.

Examples: 1, 2, 4, 8, 16,... and 81, -27, 9, -3, 1, -1/3,... r = common ratio

r = 2 r = $-\frac{1}{3}$

THE NTH TERM OF A GEOMETRIC SEQUENCE

$$a_n = a_1 \bullet r^{n-1}$$

1) Find the 16th term of $1, -\sqrt{2}, 2, \ldots$

$r = -\sqrt{2}$

$a_{16} = 1 \cdot (-\sqrt{2})^{15}$

$\boxed{a_{16} = -2^{15/2}}$

2) Find the 30th term of $2, -4, 8, \ldots$

$r = -2$

$a_{30} = 2 \cdot (-2)^{29}$

$\boxed{a_{30} = -2^{30}}$

3) Find the 5th term of a geometric sequence if the 4th term is 4 and the 6th term is 6.

$a_4 = 4 \quad a_6 = 6 \quad a_n = a_1 \cdot r^{n-1}$

$4 = a_1 \cdot r^3 \qquad a_1 = \frac{4}{r^3}$

$6 = a_1 \cdot r^5$

$\therefore \; 6 = \frac{4}{r^3} \cdot r^5$

$6 = 4r^2$

$r = \pm\sqrt{\frac{3}{2}}$

$\boxed{\text{5th term: } 4\sqrt{\tfrac{3}{2}} \text{ or } -4\sqrt{\tfrac{3}{2}}}$

4) If the fourth term in a geometric sequence is $\frac{4}{3}$ and the 7th term is $\frac{32}{81}$ find the first term.

$a_4 = \frac{4}{3} \quad a_7 = \frac{32}{81}$

$\frac{4}{3} = a_1 \cdot r^3 \qquad a_1 = \frac{4}{3r^3}$

$\frac{32}{81} = a_1 \cdot r^6$

$\frac{32}{81} = \frac{4}{3r^3} \cdot r^6$

$\frac{32}{81} = \frac{4}{3} r^3$

$\frac{8}{27} = r^3$

$r = \sqrt[3]{8/27} = \frac{2}{3}$

$a_1 = \frac{4}{3r^3}$

$a_1 = \frac{4}{3(\frac{2}{3})^3}$

$a_1 = \frac{4}{3(8/27)} = \frac{4}{8/9}$

$a_1 = \boxed{9/2}$

PARTIAL SUM OF A GEOMETRIC SERIES: $S_n = \dfrac{a_1(1-r^n)}{1-r}$ (sum of a finite series)

Find the indicated sum.

5) S_5 for $1 + 2 + 4 + \ldots$

$$S_5 = \frac{1(1-2^5)}{1-2} = \frac{1(-31)}{-1} = \boxed{31}$$

6) S_6 with $a_1 = 10$ and $a_2 = 8$

$r = \frac{4}{5}$

$$S_6 = 10\left(\frac{1-\left(\frac{4}{5}\right)^6}{1-\frac{4}{5}}\right)$$

$$10 \cdot \frac{5}{1}\left(1-\left(\frac{4}{5}\right)^6\right) = 50 - 50 \cdot \frac{4^6}{5^6}$$

7) Find the sum of the first 5 terms. $-\frac{1}{2}, \frac{3}{10}, -\frac{9}{50}, \ldots$

$r = -\frac{3}{5}$

$$S_5 = -\frac{1}{2}\left(\frac{1-\left(-\frac{3}{5}\right)^5}{1+\frac{3}{5}}\right) = -\frac{1}{2}\left(\frac{1+\frac{3^5}{5^5}}{\frac{8}{5}}\right)$$

$$= \frac{-5}{16}\left(1+\frac{243}{3125}\right)$$

$$= \boxed{\frac{-421}{1250}}$$

8) Find $\displaystyle\sum_{k=1}^{6}\left(\frac{2}{3}\right)^{k+1}$

$a_1 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$$S_6 = \frac{4}{9}\left(\frac{1-\left(\frac{2}{3}\right)^6}{1-\frac{2}{3}}\right)$$

$$= \frac{4}{9}\left(\frac{1-\frac{2^6}{3^6}}{\frac{1}{3}}\right) = \boxed{\frac{2660}{2187}}$$

Special Case: If the **common ratio** is between –1 and 1, an infinite geometric sequence has a finite sum.

SUM OF AN INFINITE GEOMETRIC SERIES: $S = \dfrac{a_1}{1-r}$

Find the sum of the infinite geometric series:

9) $1 + \frac{2}{3} + \frac{4}{9} + \ldots$

$$S = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \boxed{3}$$

10) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+1}$

$$S = \frac{\frac{4}{9}}{1-\frac{2}{3}} = \frac{4/9}{1/3} = \boxed{\frac{4}{3}}$$

11) $1 + \frac{1}{1.01} + \frac{1}{(1.01)^2} + \ldots$

$\boxed{r = \frac{1}{1.01}}$

$$S = \frac{1}{1-\frac{1}{1.01}} = \frac{1}{\frac{1}{101}} = \boxed{101}$$

12) $-1 - \frac{1}{\sqrt{2}} - \frac{1}{2} - \ldots$

$r = \frac{1}{\sqrt{2}}$

$$S = \frac{-1}{1-\frac{1}{\sqrt{2}}} = \frac{-1}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{-1}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{-\sqrt{2}}{\sqrt{2}-1}$$

$$\frac{-\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{-2-\sqrt{2}}{2-1} = \boxed{-2-\sqrt{2}}$$