QUADRATIC EQUATIONS

OBJECTIVES: 1) Use the quadratic formula to find the roots of an equation.
2) Compute the sum and product of the roots.

DEFINITION: ROOT

A function **root** is a solution to the equation, an x-intercept and the location where the function's graph crosses the x-axis.

THE PRODUCT AND SUM OF ROOTS THEOREM:

a=1!

For quadratics of the form $x^2 + bx + c = 0$, roots r_1 and r_2 have the following properties:

$r_1 \bullet r_2 = c$ and $r_1 + r_2 = -b$

Examples:

1) Find a quadratic function with integer coefficients whose roots are $\frac{2-\sqrt{3}}{5}$ and $\frac{2+\sqrt{3}}{5}$.

$r_1 \cdot r_2 = c$

$c = \left(\frac{2-\sqrt{3}}{5}\right)\left(\frac{2+\sqrt{3}}{5}\right)$

$c = \frac{4-3}{25} = \frac{1}{25}$

$r_1 + r_2 = -b$ or $-(r_1+r_2) = b$

$-b = \frac{2+\sqrt{3}}{5} + \frac{2-\sqrt{3}}{5}$

$-b = \frac{4}{5}$ $b = -4/5$

$y = x^2 - \frac{4}{5}x + \frac{1}{25}$ → we need integer coefficients, so multiply by 25

$\boxed{y = 25x^2 - 20x + 1}$

2) Find the product and sum of roots for each.

a. $(x-2)(x+4) = 1$ [Complete the square.]

$x^2 + 2x - 8 = 1$

$x^2 + 2x = 9$

$x^2 + 2x + 1 = 10$

$(x+1)^2 = 10$

$x = -1 \pm \sqrt{10}$

Sum:

$-1 + \sqrt{10} + (-1 - \sqrt{10})$

$\boxed{-2}$

Product:

$(-1+\sqrt{10})(-1-\sqrt{10})$

$1 - 10$

$\boxed{-9}$

b. $-5y^2 + 10y = -2$ [Use quadratic formula.]

$-5y^2 + 10y + 2 = 0$

$y = \frac{-10 \pm \sqrt{100 - 4(-5)(2)}}{2(-5)}$

$y = \frac{-10 \pm \sqrt{140}}{-10}$

$y = \frac{-10 \pm 2\sqrt{35}}{-10}$

$y = \frac{5 \pm \sqrt{35}}{5}$

Sum:

$\left(\frac{5+\sqrt{35}}{5}\right) + \left(\frac{5-\sqrt{35}}{5}\right)$

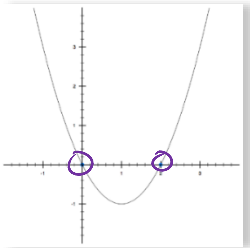
$\boxed{2}$

Product:

$\left(\frac{5+\sqrt{35}}{5}\right)\left(\frac{5-\sqrt{35}}{5}\right)$

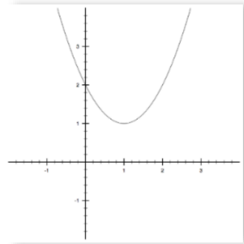
$\frac{25-35}{25} = \boxed{\frac{-2}{5}}$

DISCRIMINANT REVIEW



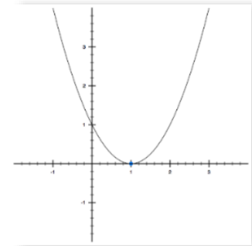
2 real solutions

$b^2-4ac>0$



No real solutions, but 2 complex

$b^2-4ac<0$



1 real sol, double root

$b^2-4ac=0$

3)How many roots? $\frac{\sqrt{3}x^2}{6}+\frac{\sqrt{2}x}{3}-\frac{3}{4}=0$

Check:

$b^2-4ac = \left(\frac{\sqrt{2}}{3}\right)^2-4\left(\frac{\sqrt{3}}{6}\right)\left(-\frac{3}{4}\right)$

$= \underbrace{\frac{2}{9}+\frac{\sqrt{3}}{2}}$

positive, ∴ 2 real roots

4) Find k such that there is exactly 1 root.

$$5x^2+(\sqrt{3k})x+4=0$$

If only 1 root, $b^2-4ac=0$

$(\sqrt{3k})^2-4(5)(4)=0$

$3k-80=0$

$\frac{80}{3}=k$

5) Determine the number of real roots the equation has:

a) $x^2-3x+6=0$

b^2-4ac

$9-4(1)(6)=$ − number

$\boxed{\text{No real roots}}$

b) $-3x^2-x+2=0$

b^2-4ac

$(-1)^2-4(-3)(2)$

$1+24=25$

$\boxed{\text{2 real roots}}$