2.2
Notes

OTHER TYPES OF EQUATIONS: PART 1

OBJECTIVES: 1) Solve absolute value and radical equations and equations using nth roots.
2) Solve equations of quadratic type.

REVIEW

When is $|x-4|$ rewritten as $x-4$ and when is $|x-4|$ rewritten as $-(x-4)$?

$|x-4|$ is $x-4$ if $x-4 \geq 0$ (in other words, if inside is positive or zero)
and is $-(x-4)$ if $x-4 < 0$ (in other words, if inside is negative)

ABSOLUTE VALUE EQUATIONS

1) Solve $|5x-3| = 12$

If $5x-3$ is positive, then:

$5x-3=12$

$5x=15$

$x=3$

If $5x-3$ is neg. then:

$-(5x-3)=12$

$-5x+3=12$

$-5x=9$

$x=-9/5$

2) $|x^2+5x| = |3x+16|$

Either $x^2+5x=3x+16$ or they are opposites:

$x^2+5x=3x+16$

$x^2+2x-16=0$

$x=\dfrac{-2\pm\sqrt{4-4(1)(-16)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{68}}{2}$

$\boxed{x=-1\pm\sqrt{17}}$

$x^2+5x=-(3x+16)$

$x^2+5x=-3x-16$

$x^2+8x+16=0$

$(x+4)^2=0$

$\boxed{x=-4}$

3) Solve $|5x-3| = -12$

No solution!

Abs. value of $5x-3$ must be positive!

USING NTH ROOTS TO SOLVE EQUATIONS

nth roots
If n is **even** then possibly 2 answers (or 1 or none).
If n is **odd**, always 1 solution.

2) $(x-1)^4 = 7$

$\sqrt[4]{(x-1)^4}=\sqrt[4]{7}$

$x-1=\pm\sqrt[4]{7}$

$\boxed{x=1\pm\sqrt[4]{7}}$

2 real solutions

3) $(x-1)^4 = -7$

$\sqrt[4]{(x-1)^4}=\sqrt[4]{-7}$

$x-1=\pm\sqrt[4]{-7}$

not real!

no real solutions

4) $3(x-1)^5 = -48$

$(x-1)^5=-16$

$\sqrt[5]{(x-1)^5}=\sqrt[5]{-16}$

$x-1=\sqrt[5]{-16}$

$\boxed{x=1+\sqrt[5]{-16}}$

only 1 real solution here

RADICAL EQUATIONS

This is a BEAST!

5) $\sqrt{2y-3}-\sqrt{3y+3}+\sqrt{3y-2}=0$

$\left(\sqrt{2y-3}+\sqrt{3y-2}\right)^2=\left(\sqrt{3y+3}\right)^2$ (isolate & square both sides)

$2y-3+2\sqrt{(2y-3)(3y-2)}+3y-2=3y+3$

$2\sqrt{(2y-3)(3y-2)}=-2y+8$ (simplify)

$\left(\sqrt{(2y-3)(3y-2)}\right)^2=(-y+4)^2$ (square both sides)

$(2y-3)(3y-2)=y^2-8y+16$ (simplify left side)

Solve the quadratic:

$6y^2-4y-9y+6=y^2-8y+16$

$5y^2-5y-10=0$

$5(y^2-y-2)=0$

$5(y-2)(y+1)=0$

$y=2 \quad y=-1 \longrightarrow$ now check:

$y=2$

$\sqrt{2(2)-3}-\sqrt{3(2)+3}+\sqrt{3(2)-2}=0$

$\sqrt{1}-\sqrt{9}+\sqrt{4}=0$

$1-3+2=0$

$0=0$ ✓

extraneous! $\cancel{y=-1}$

$\sqrt{2(-1)-3}-\sqrt{3(-1)+3}+\sqrt{3(-1)-2}=0$

$\sqrt{-1}$ ← not real

Solution: $\boxed{y=2}$

SOLVING EQUATIONS OF QUADRATIC TYPE

One exponent will be double the other and you will have a constant term.

7) $x^4-x^2=12$

let $x^2=t$

$t^2-t=12$

$t^2-t-12=0$

$(t-4)(t+3)=0$

$t=4 \quad t=-3$

$t=x^2$

$x^2=4 \quad x^2=-3$ ← not real

$x=\pm 2$

$\{\pm 2\}$

9) $x^4-8x^2+8=0$

let $t=x^2$

$t^2-8t+8=0$ ← not factorable!

$t=\dfrac{8\pm\sqrt{8^2-4(1)(8)}}{2(1)}$

$t=\dfrac{8\pm\sqrt{64-32}}{2}$

$t=\dfrac{8\pm\sqrt{32}}{2}$

$t=\dfrac{8\pm 4\sqrt{2}}{2}$

$t=4\pm 2\sqrt{2}$

$t=x^2$, substitution

$x^2=4\pm 2\sqrt{2}$

$x=\pm\sqrt{4\pm 2\sqrt{2}}$

4 solutions!

$x=\sqrt{4+2\sqrt{2}},\ \sqrt{4-2\sqrt{2}},\ -\sqrt{4+2\sqrt{2}},\ -\sqrt{4-2\sqrt{2}}$

10) $3x^{\frac{8}{5}}-5x^{\frac{4}{5}}-12=0$

let $t=x^{4/5}$

$3t^2-5t-12=0$

$3t^2-9t+4t-12=0$

$3t(t-3)+4(t-3)=0$

$(3t+4)(t-3)=0$

$t=-\frac{4}{3} \quad t=3$

subst. $x^{4/5}=t$

$\left(x^{4/5}\right)^{5/4}=\left(-\frac{4}{3}\right)^{5/4} \qquad \left(x^{4/5}\right)^{5/4}=(3)^{5/4}$

raise to recip. power

$x=\sqrt[4]{\left(-\frac{4}{3}\right)^5}$ ← This is NOT real!

$x=\pm\sqrt[4]{3^5}$ ← 2 real solutions!

Solution: $x=\pm\sqrt[4]{3^5}$

11) $6x^{-2}-x^{-1}-2=0$

let $t=x^{-1}$

$6t^2-t-2=0$

$6t^2-4t+3t-2=0$

$2t(3t-2)+1(3t-2)=0$

$(2t+1)(3t-2)=0$

$t=-\frac{1}{2} \quad t=\frac{2}{3}$

subst: $t=x^{-1}$

$x^{-1}=-\frac{1}{2} \qquad x^{-1}=\frac{2}{3}$

$\frac{1}{x}=-\frac{1}{2} \qquad \frac{1}{x}=\frac{2}{3}$

$x=-2 \qquad x=\frac{3}{2}$

Solution: $x=-2,\ \frac{3}{2}$