

OTHER TYPES OF EQUATIONS: PART 2

- OBJECTIVES:**
- 1) Solve literal equations, absolute value and radical equations, and equations using n^{th} roots.
 - 2) Solve equations of quadratic type.

LITERAL EQUATIONS

1) Solve for z : $w = \frac{z}{z+w}$

$$(z+w)w = \frac{z}{z+w} (z+w)$$

$$zw + w^2 = z$$

$$w^2 = z - zw$$

$$w^2 = z(1-w)$$

$$\boxed{\frac{w^2}{1-w} = z}$$

2) Solve for y in terms of the other letters:

$$xy^2 - 5y = 3x$$

$$xy^2 - 5y - 3x = 0 \quad \text{let } a = x, b = -5, c = -3x$$

$$y = \frac{5 \pm \sqrt{25 - 4(x)(-3x)}}{2(x)} = \boxed{\frac{5 \pm \sqrt{25 + 12x^2}}{2x}}$$

REVIEW:

Solve for x in each of the equations.

3) $t - t^3 = 0$

$$t(1-t^2) = 0$$

$$t(1+t)(1-t) = 0$$

$$t = 0, -1, 1$$

4) $\sqrt{3+2t} + \sqrt{-1+4t} = 1$

$$(\sqrt{3+2t})^2 = (1 - \sqrt{-1+4t})^2$$

$$3+2t = 1 - 2\sqrt{-1+4t} + -1+4t$$

$$3-2t = -2\sqrt{-1+4t}$$

$$9-12t+4t^2 = 4(-1+4t)$$

$$4t^2 - 28t + 13 = 0$$

$$4t^2 - 26t - 2t + 13 = 0$$

$$2t(2t-13) - 1(2t-13) = 0$$

$$(2t-1)(2t-13) = 0$$

$$t = \frac{1}{2} \quad t = \frac{13}{2}$$

No Solution!

Both $\frac{1}{2} \neq \frac{13}{2}$
extraneous!

check:

$$t = \frac{1}{2} \text{ NO!}$$

$$\sqrt{3+1} + \sqrt{-1+2} = 1$$

$$2 + 1 \neq 1$$

$$t = \frac{13}{2} \text{ NO!}$$

$$\sqrt{3+13} + \sqrt{-1+4(\frac{13}{2})} = 1$$

$$4 + 5 \neq 1$$

5) $(x-2)^4 + 16 = 19$

$$(x-2)^4 = 3$$

$$x-2 = \pm \sqrt[4]{3}$$

$$\boxed{x = 2 \pm \sqrt[4]{3}}$$

6) $(x-2)^4 + 19 = 16$ (not a typo!)

$$(x-2)^4 = -3$$

$$x-2 = \sqrt[4]{-3}$$

no real solutions

$$7) \frac{x-2}{x+5} = \frac{7}{x+3}$$

$$(x-2)(x+3) = 7(x+5)$$

$$x^2 + x - 6 = 7x + 35$$

$$x^2 - 6x - 41 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(-41)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{200}}{2}$$

$$x = \frac{6 \pm 10\sqrt{2}}{2}$$

$$x = 3 \pm 5\sqrt{2}$$

$$9) |x-3| = |3x+6|$$

$$x-3 = 3x+6$$

$$x-3 = -(3x+6)$$

$$2x = -9$$

$$x-3 = -3x-6$$

$$x = -\frac{9}{2}$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

Solution:

$$x = -\frac{9}{2}, -\frac{3}{4}$$

$$8) 2x^{\frac{6}{5}} - 3x^{\frac{3}{5}} - 5 = 0$$

$$\text{let } t = x^{\frac{3}{5}}$$

$$2t^2 - 3t - 5 = 0$$

$$2t^2 - 5t + 2t - 5 = 0$$

$$t(2t-5) + 1(2t-5) = 0$$

$$(t+1)(2t-5) = 0$$

$$t = -1 \quad t = \frac{5}{2}$$

substitute $t = x^{\frac{3}{5}}$

$$(x^{\frac{3}{5}})^{\frac{5}{3}} = (-1)^{\frac{5}{3}} \quad (x^{\frac{3}{5}})^{\frac{5}{3}} = \left(\frac{5}{2}\right)^{\frac{5}{3}}$$

$$x = \sqrt[3]{(-1)^5}$$

$$x = \sqrt[3]{-1}$$

$$x = -1$$

$$x = \sqrt[3]{\frac{5^5}{2^5}} \leftarrow \text{real}$$

Solution:

$$x = -1, \left(\frac{5}{2}\right)^{\frac{5}{3}} \text{ or } \sqrt[3]{\frac{5^5}{2^5}}$$

$$10) |x-3| \leq 0$$

$$|x-3| < 0 ?$$

$$|x-3| = 0$$

NO.

not possible!

$$x-3=0$$

$$x=3$$