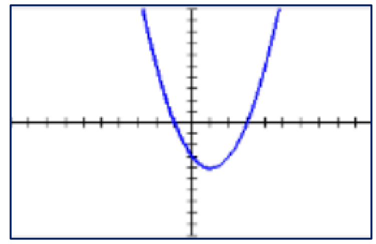
2.4
Notes

MORE ON POLYNOMIAL INEQUALITIES

OBJECTIVES: 1) Solve polynomial inequalities using a graphical approach.
2) Solve rational expression inequalities.

SOLVING INEQUALITIES: USING A GRAPHICAL APPROACH



1) $x^2 - 2x - 3 < 0$

$(-1, 3)$

2) $x^2 - 2x - 3 \geq 0$

$(-\infty, -1] \cup [3, \infty)$

KEY NUMBERS: Roots and restrictions on domain

SOLVING INEQUALITIES: WITHOUT A GRAPH

3) $x^4 \leq 14x^3 - 48x^2$

$x^4 - 14x^3 + 48x^2 \leq 0$

$x^2(x^2 - 14x + 48) \leq 0$

$x^2(x-6)(x-8) \leq 0$

$x=0 \quad x=6 \quad x=8$ } KEY #s!

$(-\infty, 0) \quad (0,6) \quad (6,8) \quad (8, \infty)$

$(+) \quad 0 \quad (+) \quad 6 \quad (-) \quad 8 \quad (+)$

$x^2(x-6)(x-8)$

$(-\infty, 0) \rightarrow$ test

$(+)(-)(-) = (+)$

$(0,6)$: $(+)(-)(-) = (+)$

$(6,8)$: $(+)(+)(-) = (-)$

$(8,0)$: $(+)(+)(+) = (+)$

Polynomial ≤ 0 at $(6,8)$ AND when $x=0$

Solution: $(6,8) \cup \{0\}$

4) $x^3 - 2x^2 - 3x > 0$

$x(x^2 - 2x - 3) > 0$

$x(x-3)(x+1) > 0$

$x = 0, 3, -1$ ← key #s

$(-\infty, -1) \; (-1, 0) \; (0, 3) \; (3, \infty)$

$(-) \quad -1 \quad (+) \quad 0 \quad (-) \quad 3 \quad (+)$

TEST: $x(x-3)(x+1)$

$(-\infty, -1)$: $(-)(-)(-) = (-)$

$(-1, 0)$: $(-)(-)(+) = (+)$

$(0, 3)$: $(+)(-)(+) = (-)$

$(3, \infty)$: $(+)(+)(+) = (+)$

Polynomial is > 0 at these intervals.

Solution: $(-1, 0) \cup (3, \infty)$

RATIONAL EXPRESSION INEQUALITIES:

5) $\dfrac{x+2}{x-4} \ge 0$

Key #s: $-2, 4$

$(-\infty,-2)\ (-2,4)\ (4,\infty)$

$(+)\ {-2}\ (-)\ 4\ (+)$

Test: $\dfrac{x+2}{x-4}$

$(-\infty,-2)$: $\dfrac{(-)}{(-)} = (+)$

$(-2,4)$: $\dfrac{(+)}{(-)} = (-)$

$(4,\infty)$: $\dfrac{(+)}{(+)} = (+)$

Solution: $(-\infty,-2] \cup (4,\infty)$

6) $\dfrac{1}{x-2} - \dfrac{1}{x-1} \ge \dfrac{1}{6}$

You may want to mult. by $(x-2)(x-1)$, but we don't know if it is a pos/neg. value!

$\dfrac{1}{x-2} - \dfrac{1}{x-1} - \dfrac{1}{6} \ge 0$

$\dfrac{6(x-1) - 6(x-2) - (x-2)(x-1)}{6(x-2)(x-1)} \ge 0$

$\dfrac{6x-6-6x+12-(x^2-3x+2)}{6(x-2)(x-1)} \ge 0$

$\dfrac{-x^2+3x+4}{6(x-2)(x-1)} \ge 0$

$\dfrac{-(x^2-3x-4)}{6(x-2)(x-1)} \ge 0$

$\dfrac{-(x-4)(x+1)}{6(x-2)(x-1)} \ge 0$

Key #s: $4, -1, 2, 1$

$(-\infty,-1)\ (-1,1)\ (1,2)\ (2,4)\ (4,\infty)$

$-1\quad 1\quad 2\quad 4$

Solution: $[-1,1) \cup (2,4]$

TEST: $\dfrac{-(x-4)(x+1)}{6(x-2)(x-1)}$

$(-\infty,-1)$: $\dfrac{(-)(-)(-)}{(-)(-)} = (-)$

$(-1,1)$: $\dfrac{(-)(-)(+)}{(-)(-)} = (+)$

$(1,2)$: $\dfrac{-(-)(+)}{(-)(+)} = (-)$

$(2,4)$: $\dfrac{(-)(-)(+)}{(+)(+)} = (+)$

$(4,\infty)$: $\dfrac{(-)(+)(+)}{(+)(+)} = (-)$

7) $\dfrac{2x+1}{x-1} - \dfrac{2}{x-3} \le 1$

$\dfrac{2x+1}{x-1} - \dfrac{2}{x-3} - 1 \le 0$

$\dfrac{(2x+1)(x-3) - 2(x-1) - (x-1)(x-3)}{(x-1)(x-3)} \le 0$

$\dfrac{2x^2-5x-3-2x+2-(x^2-4x+3)}{(x-1)(x-3)} \le 0$

$\dfrac{x^2-3x-4}{(x-1)(x-3)} \le 0$

$\dfrac{(x-4)(x+1)}{(x-1)(x-3)} \le 0$

Key #s: $1, 3, -1, 4$

$(-\infty,-1)\ (-1,1)\ (1,3)\ (3,4)\ (4,\infty)$

$(+)\ {-1}\ (-)\ 1\ (+)\ 3\ (-)\ 4\ (+)$

Test: $\dfrac{(x-4)(x+1)}{(x-1)(x-3)}$

$(-\infty,-1)$: $\dfrac{(-)(-)}{(-)(-)} = (+)$

$(-1,1)$: $\dfrac{(-)(+)}{(-)(-)} = (-)$

$(1,3)$: $\dfrac{(-)(+)}{(+)(-)} = (+)$

$(3,4)$: $\dfrac{(-)(+)}{(+)(+)} = (-)$

$(4,\infty)$: $\dfrac{(+)(+)}{(+)(+)} = (+)$

Solution: $[-1,1) \cup (3,4]$

closed $x=-1,4$ make numer. $=0$

open b/c denom can't $=0$