3.1–3.6 REVIEW

Section 3.1

Find the domain of the function:

1. $y=\sqrt{\dfrac{2x-3}{3x+2}}$ D: $\left(-\infty, \frac{-2}{3}\right) \cup \left[\frac{3}{2}, \infty\right)$

$\frac{2x-3}{3x+2} \geq 0$

$+ \quad \frac{-2}{3} \quad - \quad \frac{3}{2} \quad +$

Test: $\left(-\infty, \frac{-2}{3}\right)$: $\frac{-}{-} = +$

$\left(\frac{-2}{3}, \frac{3}{2}\right) = \frac{-}{+} = -$

$\left(\frac{3}{2}, \infty\right) = \frac{+}{+} = +$

Find the range of the function:

2. $f(x)=\sqrt{2x^3-5}$

$y = \sqrt{2x^3-5}$

$y^2 = 2x^3-5$

$\sqrt[3]{\frac{y^2+5}{2}} = x$

This will always be positive!

$y \geq 0$

4. If $f(x)=2x-4$, solve for x if $f(x+1)=f\left(\dfrac{1}{x}\right)$

$2(x+1)-4 = 2\left(\frac{1}{x}\right)-4$

$2x+2-4 = \frac{2}{x}-4$

$2x+2 = \frac{2}{x}$

$2x^2+2x-2=0$

$2(x^2+x-1)=0$

$\frac{-1 \pm \sqrt{1^2-4(1)(-1)}}{2(1)} = x$

$\boxed{\frac{-1 \pm \sqrt{5}}{2} = x}$

Section 3.2 and 3.3

5) Given $f(x)=\begin{cases} -x^2 & if \quad -2 \leq x < 4 \\ |x| & if \quad x \geq 4 \end{cases}$, find the average rate of change on the interval [-1,4].

$f(-1) = -1$

$f(4) = 4$

$(-1,-1)$

$(4,4)$

$\frac{4 - -1}{4 - -1} = \frac{5}{5} = \boxed{1}$

6) Find $\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-x+2$.

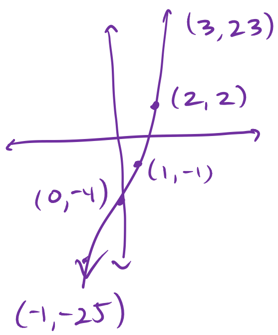
$\frac{(x+h)^2-(x+h)+2-(x^2-x+2)}{h}$

$\frac{\cancel{x^2}+2xh+h^2-\cancel{x}-h+\cancel{2}-\cancel{x^2}+\cancel{x}-\cancel{2}}{2} = \frac{2xh+h^2-h}{h} = \boxed{2x+h-1}$

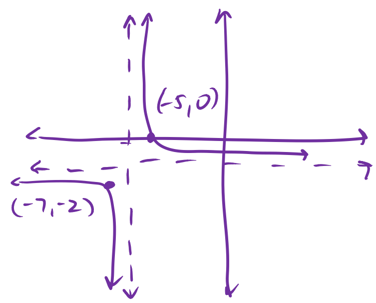
Section 3.4

7) Graph using translations. A sketch is fine!

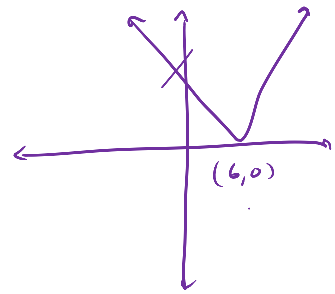
a) $y = 3(x-1)^3 - 1$



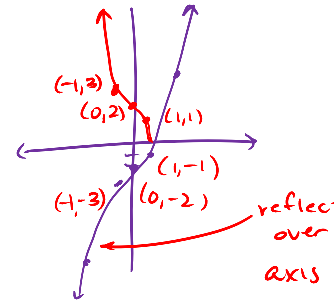
b) $y = \dfrac{1}{x+6} - 1$



c) $y = |x-6|$



d) $y = |x^3 - 2|$



8) If (3, 6) is on the graph of f(x), find the coordinates of the point on the graph of $-2f(2-x)-4$

$(3,6)$ $(3,-12)$

$(-3,-12)$

$(-1,-12)$

$\boxed{(-1,-16)}$

Section 3.5

9) If $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$ find $(f \circ g)(3)$. Also, find the domain of $(f \circ g)(x)$.

$f(g(3)) = f(\sqrt{3}) = \boxed{6}$

$f(g(x)) = \sqrt{x}^2 + 3 = x + 3$

Domain is $[0, \infty)$

10) Express $h(x) = \sqrt[4]{9+5x}$ as a composition of two simpler functions *f* and *g*, in two different ways.

$f(x) = \sqrt[4]{x}$ $f(x) = 5x$

$g(x) = 5x + 9$ $g(x) = \sqrt[4]{9+x}$

Section 3.6

11) Find the inverse of $f(x) = 3x^3 - 1$. Then prove that they are inverses.

$y = 3x^3 - 1$

$x = 3y^3 - 1$

$\sqrt[3]{\dfrac{x+1}{3}} = y$

$f(g(x)) = 3\left(\sqrt[3]{\dfrac{x+1}{3}}\right)^3 - 1$

$= 3\left(\dfrac{x+1}{3}\right) - 1$

$= x + 1 - 1$

$= \boxed{x}$

$g(f(x)) = \sqrt[3]{\dfrac{3x^3 - 1 + 1}{3}}$

$g(f(x)) = \sqrt[3]{\dfrac{3x^3}{3}}$

$g(f(x)) = \sqrt[3]{x^3} = \boxed{x}$

g & f are inverses!

12) If $f(x) = -(x+2)^3 - 4$, find $f\left(f^{-1}\left(f(-2)\right)\right)$.

$f(-2) = \boxed{-4}$