3.1
Notes

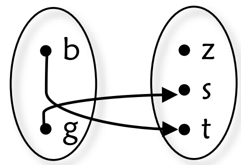
FUNCTIONS

OBJECTIVES: 1) Determine the domain and range of a function.

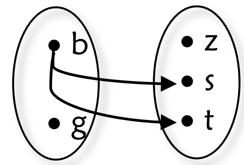
DEFINITION: FUNCTION

Let A and B be two nonempty sets. A function from A to B is a rule of correspondence that assigns to **each element in set A exactly one element in B.**

MAPPINGS



FUNCTION



NOT A FUNCTION

FUNCTION NOTATION:

Price of U.S. First-Class Stamp and the Year the Price Went into Effect							
Year (inputs)	1975	1978	1981	1985	1988	1991	1995
Price in cents (outputs)	13	15	18	22	25	29	32

1) Find f(1975) $f(1975)=13$

2) Find f(1985) – f(1978). Interpret your result.

$f(1985)=22$ $f(1978)=15$

$22-15=7$

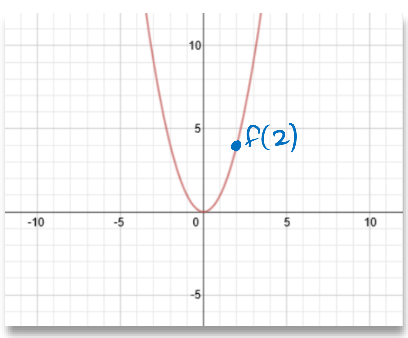
The price for a stamp increased 7 cents between 1978 and 1985.

$f(x)$

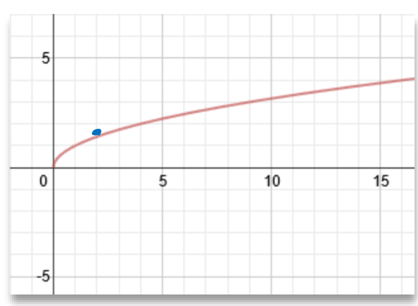
Domain: $\mathbb{R}$
$(-\infty,\infty)$

Range: $y\geq 0$

$[0,\infty)$



$h(x)$



Domain: $x\geq 0$

$[0,+\infty)$

Range: $y\geq 0$

$[0,+\infty)$

DOMAIN OF A FUNCTION: $\mathbb{R}$ unless

1) $\sqrt[even]{-\#}$ (Domain is an inequality.)

2) $\frac{\#}{0}$ (Domain excludes single numbers.)

Find the domain of the function defined by each equation:

3. a) $y=-2x+3$ $(-\infty,\infty)$

b) $y=\dfrac{1}{-2x+3}$ $(-\infty,\frac{3}{2})\cup(\frac{3}{2},\infty)$

c) $y=\sqrt{-2x+3}$ $(-\infty,\frac{3}{2}]$

d) $y=\sqrt[3]{-2x+3}$ $(-\infty,\infty)$

4. a) $y = 2x^2 - 3x - 9$ $(-\infty, \infty)$

b) $y = \dfrac{1}{2x^2 - 3x - 9}$ $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, 3) \cup (3, \infty)$

c) $y = \sqrt{2x^2 - 3x - 9}$ $(-\infty, -\frac{3}{2}] \cup [3, \infty)$

d) $y = \sqrt[3]{2x^2 - 3x - 9}$ $(-\infty, \infty)$

b) $2x^2 - 3x - 9 = 0$

$2x^2 - 6x + 3x - 9 = 0$

$2x(x-3) + 3(x-3) = 0$

$x = 3 \quad x = -\frac{3}{2}$

c) $2x^2 - 3x - 9 \geq 0$

$(-\infty, -\frac{3}{2})\ (-\frac{3}{2}, 3)\ (3, \infty)$

$+ \quad -\frac{3}{2} \quad - \quad 3 \quad +$

Test: $(2x+3)(x-3)$

$(-\infty, -\frac{3}{2})$: $(-)(-) = +$

$(-\frac{3}{2}, 3)$: $(+)(-) = -$

$(3, \infty)$: $(+)(+) = +$

5. a) $y = \dfrac{x+1}{x-2}$ $(-\infty, 2) \cup (2, \infty)$

b) $y = \sqrt{\dfrac{x+1}{x-2}}$ $(-\infty, -1] \cup (2, \infty)$

c) $y = \sqrt[3]{\dfrac{x+1}{x-2}}$ $(-\infty, 2) \cup (2, \infty)$

d) $y = \dfrac{\sqrt{5-x}}{(x-2)(x-1)}$ $(-\infty, 1) \cup (1, 2) \cup (2, 5]$

b) $\dfrac{x+1}{x-2} \geq 0$

$(-\infty, -1)\ (-1, 2)\ (2, \infty)$

$+ \quad -1 \quad - \quad 2 \quad +$

Test:

$(-\infty, -1)$: $\frac{(-)}{(-)} = +$

$(-1, 2)$: $\frac{+}{-} = -$

$(2, \infty)$: $\frac{+}{+} = +$

d) $5 - x \geq 0 \quad x \neq 2$

$x \leq 5 \quad x \neq 1$

$1 \quad 2 \quad 5$

RANGE OF A FUNCTION: Find range by either | 1) Parent function properties. (Common sense.) 2) Solve for x and examine y restrictions.

6. a) $y = 2x - 4$ $(-\infty, \infty)$

b) $y = 2x^3 - 4$ $(-\infty, \infty)$

c) $y = \dfrac{2x-5}{3x+1}$ $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

d) $y = -3 + \sqrt{4 - x^2}$ $[-3, -1]$

c) $y = \dfrac{2x-5}{3x+1}$

$y(3x+1) = 2x - 5$

$3xy + y = 2x - 5$

$3xy - 2x = -y - 5$

$x(3y-2) = -(y+5)$

$x = \dfrac{-(y+5)}{3y-2}$

d) $y = -3 + \sqrt{4 - x^2}$

$y + 3 = \sqrt{4 - x^2}$

$(y+3)^2 = 4 - x^2$

$y^2 + 6x + 9 = 4 - x^2$

$-(y^2 + 6x + 5) = x^2$

$x = \sqrt{-(y+1)(y+5)}$

$-(y+1)(y+5) \geq 0$

Test:

$(-\infty, -5)$: $-(-)(-) = -$

$(-5, -1)$: $-(-)(+) = +$

$(-1, \infty)$: $(-)(+)(+) = -$

$(-5, -1)$

not w/in range!

use $y = -3 + \sqrt{4 - x^2}$

$[-3, -1]$