3.3
Notes

SHAPES OF GRAPHS, AVERAGE RATE OF CHANGE

OBJECTIVES: 1) Find the average rate of change of a function.

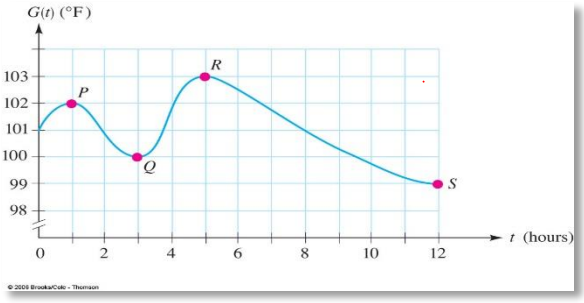
Turning Point – Graph changes from rising to falling or vice versa. P, Q, R

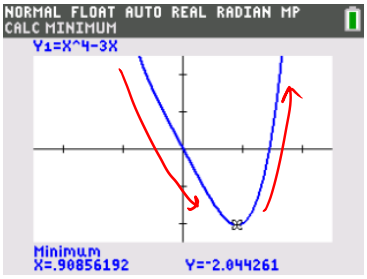
Maximum – Graph's highest point. S

Minimum – Graph's lowest point. R

Increasing – Upward trend (Q to R) (3,5)

Decreasing – Downward trend (R to S) (5,12)





FINDING MAX/MINS

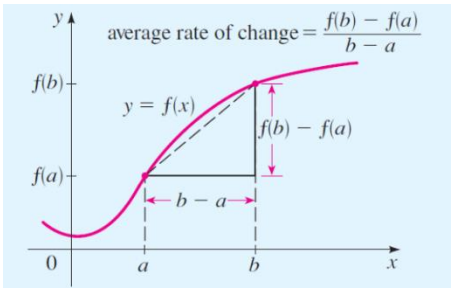
1) Find max or min and increasing/decreasing trends for $f(x) = x^4 - 3x$.

min: (.909, -2.04)

decreasing: (-∞, .909)

increasing: (.909, ∞)

AVERAGE RATE OF CHANGE: $\dfrac{\text{change in y}}{\text{change in x}} = \dfrac{f(b)-f(a)}{b-a}$



2) Find the average rate of change for $y = x^3$ on:

a) [–3,0]

f(-3) = -27 (-3,-27)
f(0) = 0 (0,0)

$\frac{-27-0}{-3-0} = 9$

b) [0,1]

f(0) = 0 (0,0)
f(1) = 1 (1,1)

$\frac{0-1}{0-1} = 1$

c) [1,3]

f(1) = 1 (1,1)
f(3) = 27 (3,27)

$\frac{27-1}{3-1} = \frac{26}{2} = 13$

3) Suppose you fell off the Sears Tower… and your position is given by the function $s(t) = -16t^2$ where t = time and s = distance. What is your average speed between 3 and 6 seconds?

s(3) = -16·9 = -144 (3,-144)
s(6) = -16·6² = -576 (6,-576)

$\frac{-576+144}{6-3} = \frac{-432}{3} = -144$

falling
-144 ft/s

3 WAYS OF SAYING THE SAME THING: (AVERAGE RATE OF CHANGE)

THE DIFFERENCE QUOTIENT AND AVERAGE VELOCITY

4) Find $\frac{f(x+h)-f(x)}{h}$ for $f(x)=3x^2-5x$.

$$\frac{3(x+h)^2-5(x+h)-(3x^2-5x)}{h}$$

$$\frac{3x^2+6xh+3h^2-5x-5h-3x^2+5x}{h}$$

$$\frac{6xh+3h^2-5h}{h}$$

$$\frac{h(6x+3h-5)}{h}$$

$$6x+3h-5$$

5) If $d(t)=-9.8t^2$ find the average velocity over [1, 1+h].

$d(1)=-9.8(1)^2=-9.8$

$d(1+h)=-9.8(1+h)^2=-9.8(1^2+2h+h^2)$

$(1,-9.8)\ (1+h,\ -9.8-19.6h-9.8h^2)$

$$\frac{-9.8-19.6h-9.8h^2-(-9.8)}{(1+h)-1}$$

$$\frac{-19.6h-9.8h^2}{h}=\frac{-h(19.6+9.8h)}{h}$$

$-(19.6+9.8h)$ ft/sec

6) Find $\frac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-5x+1$.

$$\frac{(x+h)^2-5(x+h)+1-(x^2-5x+1)}{h}$$

$$\frac{x^2+2xh+h^2-5x-5h+1-x^2+5x-1}{h}$$

$$\frac{2xh+h^2-5h}{h}$$

$$\frac{h(2x+h-5)}{h}$$

$$2x+h-5$$

7) Find $\frac{f(x)-f(a)}{x-a}$ for $f(x)=\frac{1}{3x}$.

$$\frac{\frac{1}{3x}-\frac{1}{3a}}{x-a}$$

$$\frac{\frac{a-x}{3ax}}{x-a}$$

$$\frac{a-x}{3ax}\cdot\frac{1}{x-a}$$

$$\frac{-(x-a)}{3ax}\cdot\frac{1}{x-a}=-\frac{1}{3ax}$$