COMBINING FUNCTIONS

OBJECTIVES: 1) Combine functions arithmetically.
2) Find the composition of two functions.

RULES:

- Sum: $(f+g)(x) = f(x) + g(x)$
- Difference: $(f-g)(x) = f(x) - g(x)$
- Product: $(fg)(x) = f(x) \bullet g(x)$
- Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

If $f(x) = 2x+5$ and $g(x) = x-1$ find:

1) $(f+g)(x)$

$f(x) + g(x) = 2x+5 + x-1$

$\boxed{3x+4}$

2) $(f-g)(x)$

$f(x) - g(x)$

$2x+5-(x-1)$

$\boxed{x+6}$

3) $(fg)(x)$

$f(x) \cdot g(x)$

$(2x+5)(x-1)$

$2x^2+3x-5$

4) $\left(\frac{f}{g}\right)(x)$

$\frac{f(x)}{g(x)} = \frac{2x+5}{x-1}$

COMPOSITION: Use the output from one function as the input for another.

$$(f \circ g)(x) = f(g(x))$$

If $f(x) = x+2$ and $g(x) = x-1$ find:

To simplify: 1) Plug x into g.
2) Use the result from step 1 in f.

5) $(f \circ g)(3)$

$f(g(3))$

Find $g(3)$: $g(3) = 3-1 = 2$

$f(2) = 2+2 = 4$

$\boxed{f(g(3)) = 4}$

6) $(g \circ f)(3)$

$g(f(3))$

$g(5) = \boxed{4}$

7) $(f \circ f)(3)$

$f(f(3)) = f(5) = \boxed{7}$

8) $(f \circ g)(x)$

$f(g(x)) = f(x-1) = x-1+2$

$\boxed{x+1}$

9) $(g \circ f)(x)$

$g(f(x)) = g(x+2) = x+2-1$

$\boxed{x+1}$

DOMAIN OF A COMPOSITION FUNCTION: Finding the domain of $(f \circ g)(x) = f(g(x))$

10) Find the domain of $(f \circ g)(x)$ if $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$

$f(g(x)) = x^2 + 1$

$f(g(x)) = (\sqrt{x})^2 + 1$

$f(g(x)) = x + 1$ ← suggests $\mathbb{R}$, BUT the inputs must also be the inputs (within the domain) of $g(x)$

$x \geq 0 \rightarrow$ $\boxed{D: [0, \infty)}$

11) Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{3x-4}{3x+3}$ and $g(x) = \dfrac{x+1}{x-1}$

$$f(g(x)) = \frac{3\left(\frac{x+1}{x-1}\right) - 4}{3\left(\frac{x+1}{x-1}\right) + 3} \cdot \frac{(x-1)}{(x-1)} = \frac{3(x+1) - 4(x-1)}{3(x+1) + 3(x-1)} = \frac{-x+7}{6x}$$

Domain: $\mathbb{R} \rightarrow$ but not 0

And also not 1!

$\boxed{\text{Domain: } (-\infty, 0) \cup (0, 1) \cup (1, \infty)}$

DECOMPOSING FUNCTIONS

1) Express $m(x) = (x^2 + 2)^3$ as a composition of two simpler functions *f* and *g*, in two different ways.

$f(x) = x^3$

$g(x) = x^2 + 2$

or

$f(x) = (x+2)^3$

$g(x) = x^2$

2) Express $h(x) = \sqrt[3]{3 - x^2}$ as a composition of two simpler functions *f* and *g*, in two different ways.

$f(x) = \sqrt[3]{x}$

$g(x) = 3 - x^2$

OR

$f(x) = \sqrt[3]{3 - x}$

$g(x) = x^2$