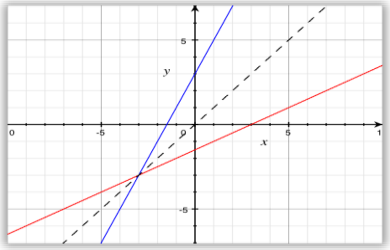
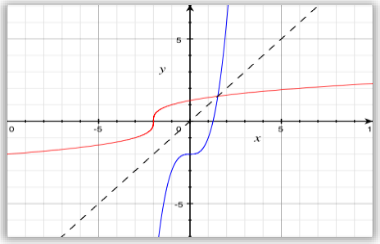
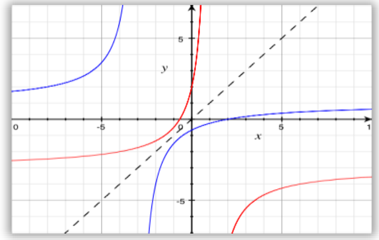
INVERSE FUNCTIONS

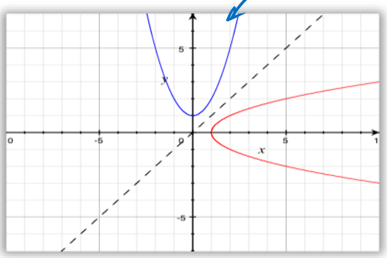
Objectives: 1) Find the inverse of a function.
2) Prove that two functions are inverses of one another.

INVERSES:
A function must be 1 to 1 in order for it to have an inverse. Rather than the vertical line test, use the horizontal line test to test for 1 to 1.









Not one to one!
Does not have an inverse function.

PROVING TWO FUNCTIONS ARE INVERSES:
Two functions f and g are inverses of one another if and only if: $f\left(g\left(x\right)\right)=x$ and $g\left(f\left(x\right)\right)=x$

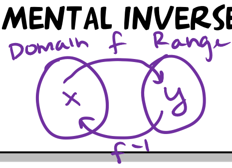
1) Prove that f and g are inverses for $f(x)=\frac{1}{2}x-4$ and $g(x)=2x+8$.

$f(g(x))=\frac{1}{2}(2x+8)-4=x+4-4=x$

$g(f(x))=2\left(\frac{1}{2}x-4\right)+8=x-8+8=x$

$g(f(x))=x$ & $f(g(x))=x$, ∴ f & g are inverses.

FUNDAMENTAL INVERSE IDENTITIES: For f, a one-to-one function, and f^{-1}, its inverse function:

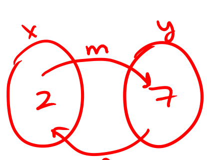


$f[f^{-1}(y)]=y$, for y in the domain f^{-1} and
$f^{-1}[f(x)]=x$, for x in the domain f

2) If m and n are inverses and $m(2) = 7$, find $n(7)$.

$m(n(7))=2$

$n(7)=2$



3) Given $f(x)=2x^3-3$ find $f[f^{-1}(4)]$
(assume that f^{-1} exists and 4 is in its domain)

$f(f^{-1}(4))=4$! (inverse identity)
No need to use (or find) f^{-1} or f.

Given a function f(x), the inverse is denoted $f^{-1}(x)$.

NOTE: $f^{-1}(x)$ does NOT mean $\dfrac{1}{f(x)}$ and the inverse of f(x) is NOT $\dfrac{1}{f(x)}$.

4) Solve for x: $f^{-1}(x-2)+5=7$ and $f(2)=10$

$f^{-1}(x-2)=2$

Equivalent!

$f(2)=10$

$\therefore\ f^{-1}(10)=2$

$x-2=10$

$\boxed{x=12}$

FINDING POSSIBLE INVERSES

1. Use x and y notation.
2. Exchange x and y.
3. Solve for y.
4. Replace $f^{-1}(x)$ notation

5) Find the inverse of $f(x)=2x^3-3$

$y=2x^3-3$

$x=2y^3-3$

$x+3=2y^3$

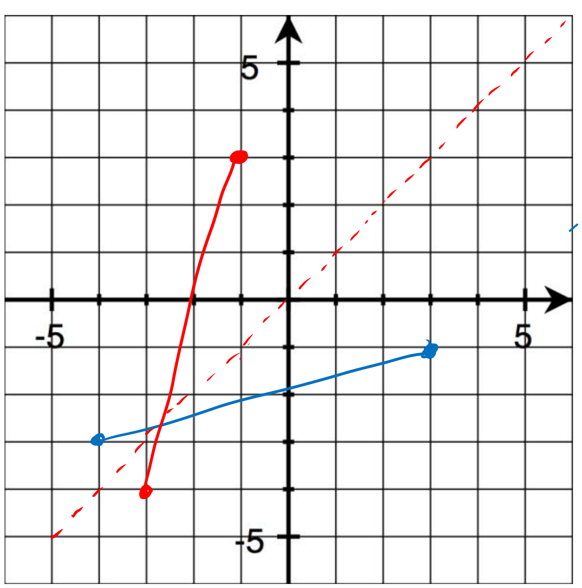
$\dfrac{x+3}{2}=y^3$

$\sqrt[3]{\dfrac{x+3}{2}}=y$

$\boxed{f^{-1}(x)=\sqrt[3]{\dfrac{x+3}{2}}}$

(−1, 3) (−3, −4)

6) $f(x)$ contains (3, –1) and (–4, –3). Sketch $f^{-1}(x)$.



(2, 8)

7) $f(x) = x^3$. Find $f^{-1}(x)$ and sketch.