4.2
Notes

QUADRATIC FUNCTIONS

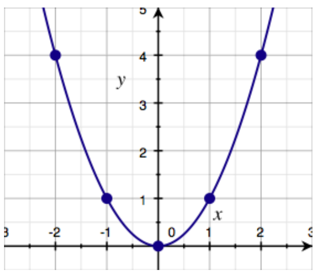
Objectives: 1) Graph quadratic functions in vertex form.
2) Complete the square to transform a quadratic equation into vertex form.
3) Find the maximum or minimum values for quadratic and quadratic like functions.

Quadratic functions are given in these two forms:

STANDARD FORM
$f(x) = ax^2 + bx + c$

VS.

VERTEX FORM
$f(x) = a(x-h)^2 + k$
(h, k) is the vertex

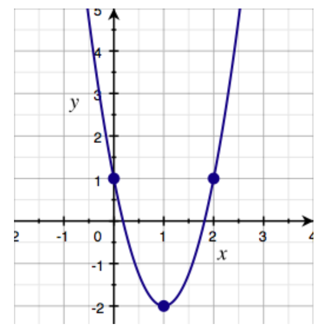


$f(x) = x^2$

a > 0 opens up
a < 0 opens down

h – axis of symmetry: x = h

|a| < 1 wide
|a| > 1 narrow



$f(x) = 3(x-1)^2 - 2$

CHANGING FROM STANDARD TO VERTEX FORM

Convert to vertex form and graph: **COMPLETE THE SQUARE!**

1) $y = -3x^2 + 12x - 8$

$y + 8 = -3(x^2 - 4x)$

$y + 8 - 12 = -3(x^2 - 4x + 4)$

$y - 4 = -3(x-2)^2$

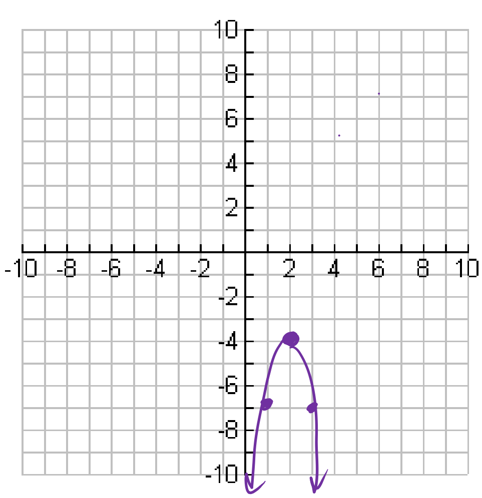
$y = -3(x-2)^2 + 4$

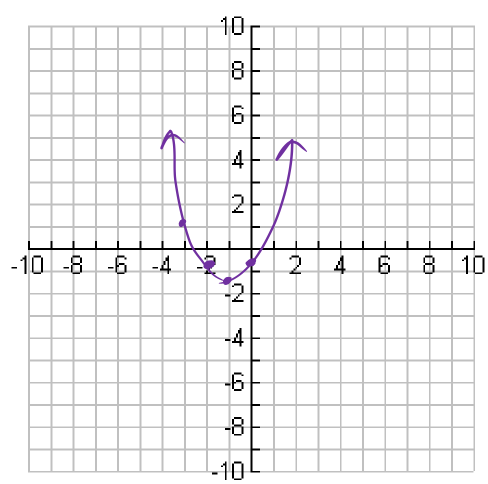
2) $y = \frac{2}{3}x^2 + \frac{4}{3}x - 1$

$y + 1 = \frac{2}{3}(x^2 + 2x)$

$y + 1 + \frac{2}{3} = \frac{2}{3}(x^2 + 2x + 1)$

$y = \frac{2}{3}(x+1)^2 - \frac{5}{3}$





FINDING MAXIMUM/MINIMUM VALUES:

The vertex is also the maximum or minimum value of a function. "The max/min, y_m occurs at x_m."

When a quadratic is in the form $f(x) = ax^2 + bx + c$, the x coordinate of the vertex is $x = \frac{-b}{2a}$.

3) Find the max/min of $f(x) = 2x^2 - 4x + 7$ min!

$\frac{-b}{2a} \rightarrow \frac{4}{2(2)} = 1$

$\boxed{(1,5) \text{ minimum}}$

$f(1) = 2(1)^2 - 4(1) + 7 = 2 - 4 + 7 = 5$

QUADRATIC-LIKE FUNCTIONS: MAX/MINS

4) Determine the input or output that produces the smallest/largest output for $f(x) = \sqrt{2x^2 - 4x + 7}$.

Input is the same!

input: 1 ← produces a minimum output

output: $f(1) = \sqrt{5}$ ← minimum output

Answer: Input is 1.

5) Find the max/min of the function: $f(x) = \sqrt[3]{2x^2 - 4x + 7}$

min: occurs at same x value!

$f(1) = \sqrt[3]{5}$

$\boxed{\text{Minimum } \sqrt[3]{5} \text{ occurs at } x = 1}$

6) Find the max/min of the function: $f(x) = \sqrt[4]{2x^2 - 4x + 7}$

min: $f(1) = \sqrt[4]{5}$

$\boxed{\text{minimum } \sqrt[4]{5} \text{ occurs at } x = 1}$