4.3 – SOLVING SYSTEMS WITH MATRICES

OBJECTIVES:

1) Solve a linear system using matrices.

Example 1)

Solve $\begin{cases} -6x + 5y = 18 \\ 7x + 2y = 26 \end{cases}$

Coefficient Matrix Variable Matrix Constant Matrix

$$\begin{bmatrix} -6 & 5 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 26 \end{bmatrix}$$

Let $[A] = \begin{bmatrix} -6 & 5 \\ 7 & 2 \end{bmatrix}$ $[B] = \begin{bmatrix} 18 \\ 26 \end{bmatrix}$

Therefore,

$$\begin{bmatrix} A \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} B \end{bmatrix}$$

Now, isolate the variable matrix.
Think about inverse operations.

INVERSE OPERATIONS!

$2x - 6 = 10$

$+6 \quad +6$

$\frac{2x}{2} = \frac{16}{2}$

mult. inverse → $\frac{1}{2} \cdot 2x = 16 \cdot \frac{1}{2}$

$x = 8$

We can't divide matrices!

$$[A]^{-1}[A]\begin{bmatrix} x \\ y \end{bmatrix} = [B][A]^{-1}$$

2×2 2×2 2×1 2×2

can't do this! Therefore, do this!

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1}[B]$$

2×2 2×1

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

POINT OF INTERSECTION:

(2, 6)

WHAT WE NEED TO SEE FOR "SHOWING WORK"

1) Define $[A]$ and $[B]$.
2) Show how you isolated the variable matrix.
3) Show your solution.
4) Write as an ordered pair.

Example 2)

Solve $\begin{cases} 2x - 3y = 16 \\ 5x + 6y = 22 \end{cases}$

1) let $[A] = \begin{bmatrix} 2 & -3 \\ 5 & 6 \end{bmatrix}$ and $[B] = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$

2) $\begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1}[B]$

3) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -1.\overline{33} \end{bmatrix}$ ← Math enter enter!

4) $\boxed{\left(6, -\frac{4}{3}\right)}$