

MODELING WITH FUNCTIONS

Objectives: 1) Model real world scenarios with functions.

SUGGESTED STEPS:

1. Picture and question
2. Define variable(s)
3. Label picture
4. Equation

GEOMETRY REVIEW!

NOTATION:

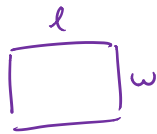
Express f in terms of x .

Examples: Express area in terms of width.
 perimeter side.
 cost units produced.
 profit number sold.

$f(x) = \dots$
 $A(w) = \dots$
 $P(s) = \dots$
 $c(u) = \dots$
 $p(n) = \dots$

EXAMPLE PROBLEMS: *No two problems are exactly alike, but you can find similarities and problem solve.

1) The perimeter of a rectangle is 50 ft. Express its area as a function of the length of a side.



$$P = 2l + 2w$$

$$A = lw$$

$$2l + 2w = 50$$

$$A(l) = l \cdot (-l + 25)$$

$$2w = -2l + 50$$

$$A(l) = -l^2 + 25l$$

$$w = -l + 25$$

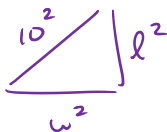
2) A rectangle is inscribed in a circle with radius 5. Express the perimeter as a function of the length.



$$P = 2l + 2w$$

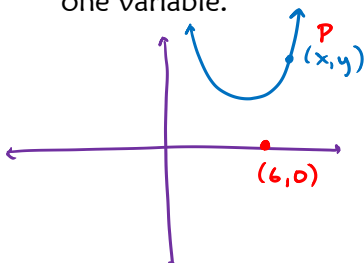
$$100 = l^2 + w^2$$

$$w = \sqrt{100 - l^2}$$



$$P(l) = 2l + 2\sqrt{100 - l^2}$$

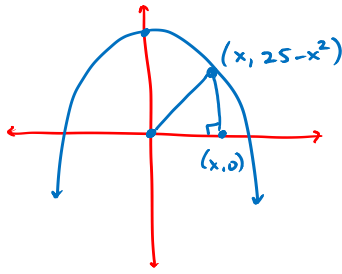
3) Let $p(x,y)$ be a point on: $y = x^2 - 4x + 9$. Express the distance from P to the point $(6,0)$ as a function of one variable.



$$D = \sqrt{(x-6)^2 + (y-0)^2}$$

$$D(x) = \sqrt{(x-6)^2 + (x^2 - 4x + 9)^2}$$

- 4) A right triangle has one vertex on $y = 25 - x^2$, one at the origin, and one at $(x, 0)$. Express the area of the triangle as a function of one variable.



$$A = \frac{1}{2}bh$$

$$A(x) = \frac{1}{2}x(25-x^2)$$

$$A(x) = \frac{25}{2}x - \frac{1}{2}x^3$$

- 5) A piece of wire x in. long is bent into a circle. Express the area in terms of x .

$$C = 2\pi r$$

$$A(x) = \pi r^2$$

$$x = 2\pi r$$

$$A(x) = \pi \left(\frac{x}{2\pi}\right)^2 = \pi \frac{x^2}{4\pi^2}$$

$$r = \frac{x}{2\pi}$$

$$A(x) = \frac{x^2}{4\pi}$$

- 6) A right circular cylinder has a volume of 10 cubic cm. Find the surface area S as a function of the radius of the base.



$$V = \pi r^2 h$$

$$\pi r^2 h = 10$$

$$h = \frac{10}{\pi r^2}$$

$$S(r) = 2\pi r^2 + 2\pi r h$$

replace h

$$S(r) = 2\pi r^2 + 2\pi r \left(\frac{10}{\pi r^2}\right)$$

$$S(r) = 2\pi r^2 + \frac{20}{r}$$

- 7) Two numbers sum to be 100. Express the product P in terms of a single number.

$$x + y = 100$$

$$y = 100 - x$$

$$P(x) = xy$$

replace y

$$P(x) = x(100 - x)$$

$$P(x) = 100x - x^2$$

- 8) Revenue = (price per unit) • (# of units sold) $R = p \cdot x$

Price is often a function of the number of units produced (demand curve): $p = m \cdot x + b$

Suppose the price (demand) curve is: $p = -0.5x + 6$. Express the revenue as a function of x

$$R(x) = p \cdot x$$

replace p

$$R(x) = (-0.5x + 6)x$$

$$R(x) = -0.5x^2 + 6x$$