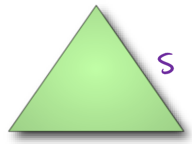
MORE MODELING

Objectives: 1) Model real world scenarios with functions.

1) A wire of length L is cut into two pieces. The first is bent into an equilateral triangle and the second into a rectangle with length twice the width. Express the total combined area as a function of *x* where *x* is the perimeter of the triangle.

$L = x + (L-x)$ ← per. of ▭

↑ not a variable - length of wire; ↑ per. of △



$P = x$

$s = \frac{x}{3}$

$L - x = 6w$

$\frac{L-x}{6} = w$

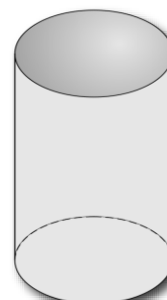
$A = \frac{\sqrt{3}}{4}s^2 + w \cdot 2w$

$A_\triangle + A_\square$

$A(x) = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + 2\left(\frac{L-x}{6}\right)^2$

$A(x) = \frac{\sqrt{3}}{36}x^2 + \frac{(L-x)^2}{18}$

2) The volume of a right circular cylinder is numerically 3 times the surface area. Find:
a) The height as a function of the radius.
b) The radius as a function of height.



V = Area of base · height $\quad SA = 2\pi r^2 + 2\pi r h$

$V = 3 \cdot SA$

$3 \cdot (\pi r^2 \cdot 2 + 2\pi r h) = \pi r^2 \cdot h$

$6\pi r^2 + 6\pi r h = \pi r^2 h$

a) $6\pi r h - \pi r^2 h = -6\pi r^2$

$h(6\pi r - \pi r^2) = -6\pi r^2$

$h = \frac{-6\pi r^2}{6\pi r - \pi r^2} = \frac{-6\pi r^2}{\pi r(6-r)} = \boxed{\frac{-6r}{6-r} = h(r)}$

b) $h = \frac{-6r}{6-r}$

$6h - rh = -6r$

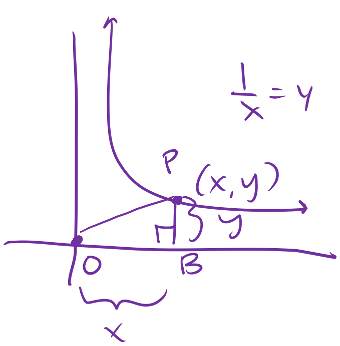
$6h = -6r + rh$

$6h = r(-6+h)$

$\boxed{r(h) = \frac{6h}{h-6}}$

3) A line is drawn from the origin *O* to a point (*x*, *y*) in the first quadrant on the graph $y = \frac{1}{x}$.

From point *P*, a line is drawn perpendicular to the *x*-axis, meeting the *x*-axis at *B*.
a) Draw a diagram for this situation.
b) Write the area of the triangle as a function of x.

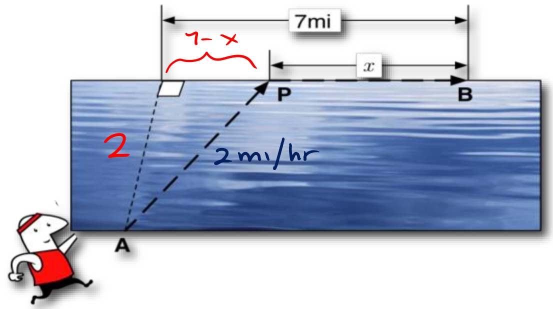


$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2}(x)(y) \quad \frac{1}{x} = y$

$A(x) = \frac{1}{2} \cdot x \cdot \frac{1}{x} = \boxed{\frac{1}{2}}$

4) A man stands at a point A on the bank of a straight river 2 mi wide. To reach point B, 7mi downstream on the opposite bank, he first rows his boat to point P on the opposite bank and then walks the remaining distance x to B. He can row at a speed of 2 mi/hr and walk at a speed of 5 mi/hr. Find a function that models the time for the trip.



$D = R \cdot T$

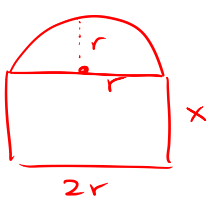
$T = \frac{D}{R}$

$D_{AP} = \sqrt{2^2 + (7-x)^2}$ $\quad R_{AP} = 2$

$D_{PB} = x$ $\quad R_{PB} = 5$

$$\boxed{T(x) = \frac{\sqrt{4 + (7-x)^2}}{2} + \frac{x}{5}}$$

5) A Norman window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 40, express the area of the window as a function of the radius of the semicircle.



$A = \frac{\pi r^2}{2} + 2r \cdot x$

replace "x"

$\pi r + 2x + 2r = 40$

$2x = -2r - \pi r + 40$

$x = -\frac{\pi r}{2} - r + 20$

or $\quad 2rx = 2xr$

$2rx = (-2r - \pi r + 40)r$

$A(r) = \frac{\pi r^2}{2} + -2r^2 - \pi r^2 + 40r$

$A(r) = \frac{\pi}{2} r^2 - 2r^2 - \pi r^2 + 40r$

$A(r) = \left(\frac{\pi}{2} - 2 - \pi\right) r^2 + 40r$

$A(r) = \left(-\frac{\pi}{2} - 2\right) r^2 + 40r$

$$\boxed{A(r) = \left(-\frac{\pi - 4}{2}\right) r^2 + 40r}$$