FINDING MAXIMA AND MINIMA

Objectives: 1) Find maximum and minimum values of a graph of a function.

MAX/MIN STRATEGY:

- One equation and one function.
- Write the function in terms of one variable only
- Then find the maximum/minimum.
- **Don't try to solve the function!**

1) **Maximizing a Product:** The sum of two numbers is one. What is the largest possible value for their product?

$x + y = 1$

$y = 1 - x$

Product:

$P = xy$

$P(x) = x(1-x)$

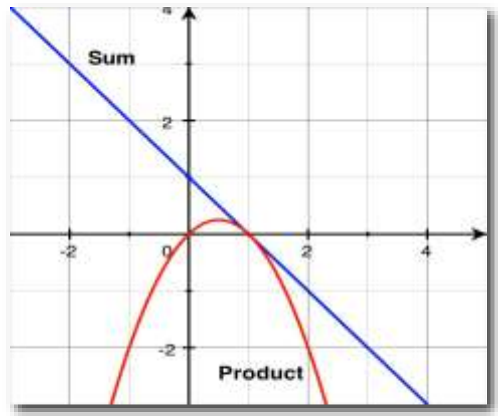
$P(x) = x - x^2$

max:

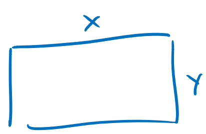
$\frac{-b}{2a} = \frac{-1}{2(-1)} = \frac{1}{2}$ $\quad f(\tfrac{1}{2}) = \frac{1}{2} - \frac{1}{4}$

$(\frac{1}{2}, \frac{1}{4})$

$\boxed{\text{LARGEST VALUE: } 1/4}$



2) **Maximizing Area:** You have 100 feet of fencing for your pig farm. What rectangular dimensions produce the largest possible area? What is that area?



$P = 2x + 2y$

$100 = 2x + 2y \rightarrow y = -x + 50$

Max:

$\frac{-b}{2a} = \frac{-50}{2(-1)} = 25$

$25 \cdot (2) + 25(2)$

$A = xy$

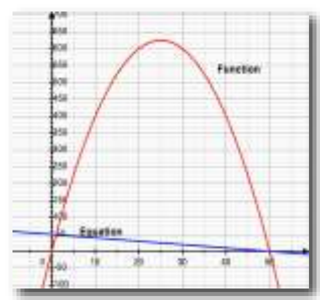
$A(x) = x(-x + 50)$

$A(x) = -x^2 + 50x$

$A(25) = -(25)^2 + 50(25)$

$A(25) = -625 + 1250$

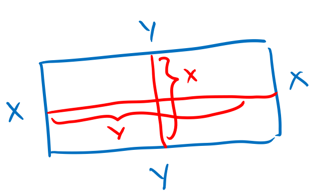
$A(25) = 625$



$\boxed{\text{Max area: } 625 \text{ ft}^2}$

Dimensions: length: 25 ft
width: 25 ft

Continued: Suppose 2 dividers are needed to create 4 pens? 2 dividers for 3 pens?



$P = 3x + 3y$

$100 = 3x + 3y \rightarrow y = -x + \frac{100}{3}$

$A(x) = x(-x + \frac{100}{3})$

$A(x) = -x^2 + \frac{100}{3}x$

Max Area: $\frac{2500}{9}$ ft^2 ≈ 278 ft^2

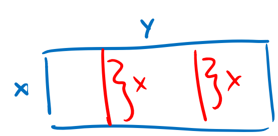
Dim: ≈ 16.7 ft by 16.7 ft

MAX:

$x = \frac{-\frac{100}{3}}{2(-1)} = \frac{-100}{-6} = \frac{50}{3}$

$A(\frac{50}{3}) = -(\frac{50}{3})^2 + \frac{100}{3} \cdot \frac{50}{3}$

$A(\frac{50}{3}) = \frac{-2500}{9} + \frac{5000}{9} = \boxed{\frac{2500}{9}}$



$P = 4x + 2y$

$100 = 4x + 2y$

$y = -2x + 50$

$A(x) = x \cdot (-2x + 50)$

$A(x) = -2x^2 + 50x$

Max Area: 312.5 ft^2

Dim: 12.5 by 25

MAX: $x = \frac{-50}{2(-2)} = \frac{25}{2}$

$A(\frac{25}{2}) = -2(\frac{25}{2})^2 + 50(\frac{25}{2}) = 312.5$

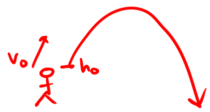
3) **Projectile motion:** $h(t) = -\frac{1}{2}at^2 + V_0 t + h_0$ and $a = \frac{32\,ft}{\sec^2}$

A 6 foot baseball player throws a ball straight up at 80 m.p.h. Find the maximum height.

$V_0 = 80$

$a = 32$

$h = 6$



$h(t) = -\frac{1}{2} \cdot 32t^2 + 80t + 6$

Max height:

$t = \frac{-80}{2(-16)} = \frac{5}{2}$ (max height occurs after 2.5 seconds)

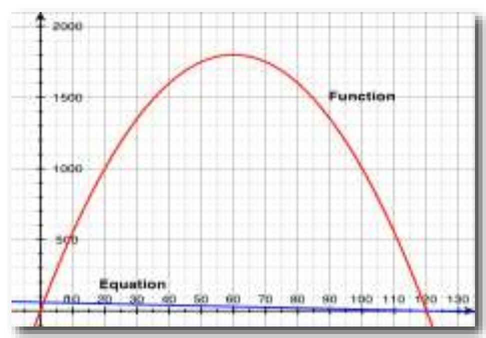
$h(\frac{5}{2}) = -16(\frac{5}{2})^2 + 80(\frac{5}{2}) + 6 = 106$

$\boxed{\text{Max height: 106 ft at 2.5 seconds}}$

4) **Economics Example:** A demand function relates price (*p*) to number of units sold (*x*). Revenue is then defined as price•number of units sold. If the price function is defined as $p(x) = -.5x + 60$.

Find both the number of units sold that maximizes revenue and the maximum revenue.

Revenue = Price · number of units sold



x = # units sold

$P(x) = -.5x + 60$

$R(x) = (-.5x + 60)\,x$

$R(x) = -.5x^2 + 60x$

Max:

$x = \frac{-60}{2(-\frac{1}{2})} = 60$

$R(60) = -.5(60)^2 + 60(60)$

$= -1800 + 3600$

$R(60) = 1800$

Max revenue is $1800 when selling 60 units