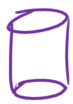
4.5
Notes
Day 2

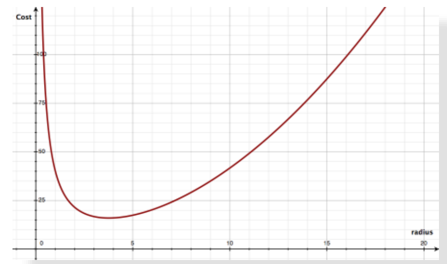
MORE MAX AND MINS

Objectives: 1) Find maximum and minimum values of a graph of a function.

1) A can in the shape of a right circular cylinder is required to have a volume of 500 cubic centimeters. The top and bottom are made of material that costs 6¢ per square centimeter, while the sides are made of material that costs 4¢ per square centimeter.

a. Express the total cost *C* of the material as a function of the radius *r* of the cylinder.
b. Use the graph to find the dimensions that make the cost *C* a minimum.





$V = \pi r^2 \cdot h$

$500 = \pi r^2 h$ ← constraint

$500 = \pi r^2 h$

$\frac{500}{\pi r^2} = h$

$SA = 2\pi r^2 + 2\pi r h$

Gives amount of material needed

$C = 2\pi r^2(.06) + 2\pi r h(.04)$

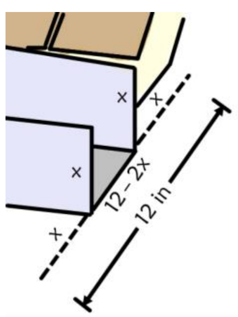
$C = .12\pi r^2 + 2\pi r \cdot \left(\frac{500}{\pi r^2}\right)(.04)$

$\boxed{C = .12\pi r^2 + \frac{40}{r}}$

Using our calculator,
Min: (3.757, 15.96)

★ Min. cost is $15.96 when radius is 3.757 cm

2) A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. What depth will provide maximum cross-sectional area and allow the most water to flow?



$A(x) = x(12-2x)$

$A(x) = 12x - 2x^2$

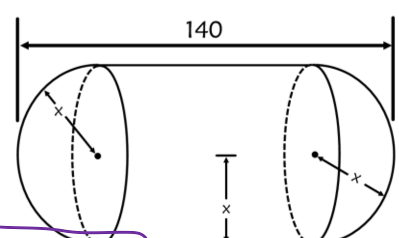
$\frac{-b}{2a} = \frac{-12}{2(-2)} = 3$ ← value for which max occurs

$\boxed{3 \times 6 \quad 3 \text{ in by } 6 \text{ in}}$

$A(3) = 12(3) - 2(3)^2$

$A(3) = 36 - 18 = 18$ ← max cross sectional area

3) A liquid storage container on a truck is in the shape of a cylinder with hemispheres on each end. The cylinders and hemispheres have the same radius. Determine the volume as a function of the radius *x*.



$140 = 2x + h$ ← $h = -2x + 140$

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$V(x) = \frac{4}{3}\pi x^3 + \pi \cdot x^2 \cdot (-2x + 140)$

$V(x) = \frac{4}{3}\pi x^3 - 2\pi x^3 + 140\pi x^2$

$\boxed{V(x) = -\frac{2}{3}\pi x^3 + 140\pi x^2}$