4.6
Notes

POLYNOMIAL FUNCTIONS

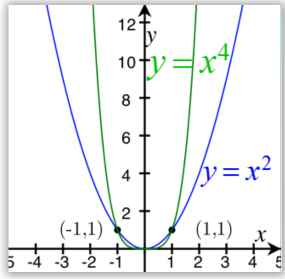
Objectives: 1) Graph polynomial functions.

POWER FUNCTION: $f(x) = kx^p$
(where k and p are constants)

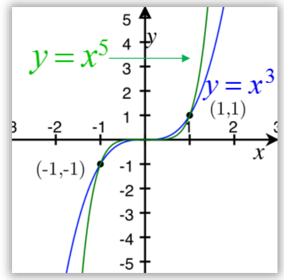
The end behavior of even integer power functions is the same.

The end behavior of odd integer functions is the same.

P = EVEN INTEGER
$y = x^2, x^4, x^6 \ldots$



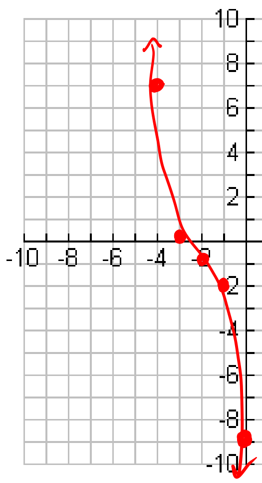
P = ODD INTEGER
$y = x^3, x^5, x^7 \ldots$



TRANSLATION EXAMPLES:

1) $y = -(x+2)^3 - 1$ $y = x^3$

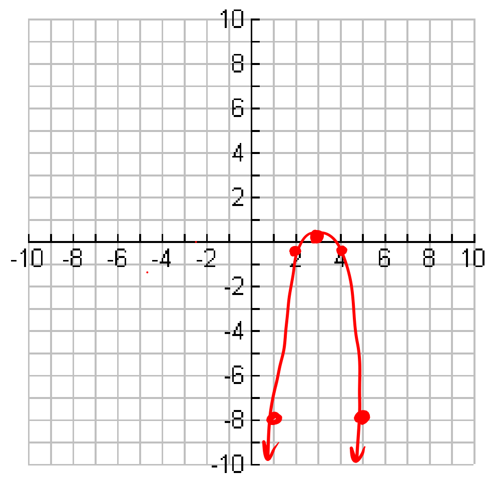
x	y
-2	-8
-1	1
0	0
1	1
2	8



reflect:
(-2,-8) is now
(-2,8)
now shift left 2
& shift down 1
(-2,8) becomes
(-4,7)

2) $y = \dfrac{-(x-3)^4}{2}$ rewrite $y = \frac{-1}{2}(x-3)^4$ $y = x^4$

x	y
-2	16
-1	1
0	0
1	1
2	16

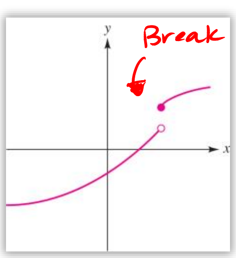


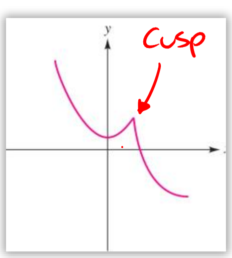
Reflect/compress:
(-2,16) is now
(-2,-8)
shift 3 right:
(-2,-8) is
now (1,-8)

POLYNOMIAL FUNCTION: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ Ex: $f(x) = x^4 - 3x^3 + 4x + 2$

PROPERTIES:
1. No breaks or cusps.
2. If degree is n, the number of **turning points** is at most n–1.
3. As x gets very + or –, then the polynomial looks like a power function.

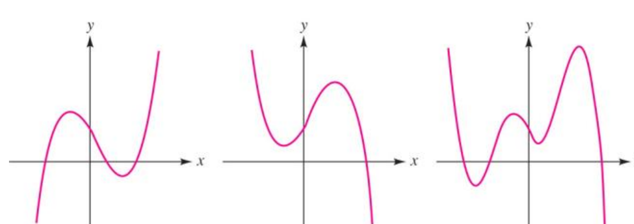
Not Polynomial Functions:





Which of the following could be the graph of $f(x) = -x^3 + x^2 + 9x + 9$?

looks like $y = -x^3$



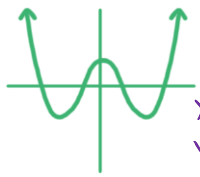
(a) (b) (c)

This looks like $y = x^3$

Too many turning points!

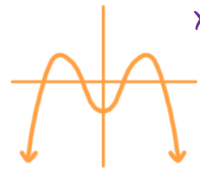
END BEHAVIOR:

DEGREE: EVEN
LEADING COEFFICIENT: +



$x \to +\infty$, $y \to +\infty$
$x \to -\infty$, $y \to +\infty$

DEGREE: EVEN
LEADING COEFFICIENT: −

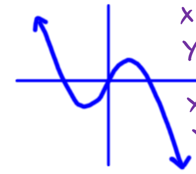


$x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to -\infty$

DEGREE: ODD
LEADING COEFFICIENT: +

$x \to +\infty$, $y \to +\infty$
$x \to -\infty$, $y \to -\infty$

DEGREE: ODD
LEADING COEFFICIENT: −

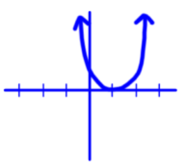


$x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to +\infty$

MULTIPLICITY: How many times a zero occurs in a function.
$f(x)=(x-2)^3$ Multiplicty 3 — The same zero

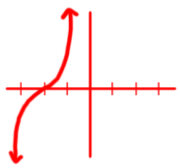
BEHAVIOR NEAR ROOTS: Determined by the exponent of the root factor and substituting that value into the other, "non-zero" terms.

EVEN
TOUCHES THE X AXIS



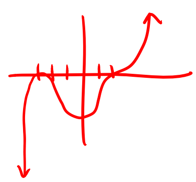
EXAMPLE: $f(x)=-(x-1)^2$

ODD
CROSSES THE X AXIS



EXAMPLE: $g(x)=(x+2)^3$

$y=x^5$



Example: $f(x)=(x-2)^3(x+3)^2$

around $x=2$,
$f(x) = (x-2)^3\cdot(5)^2 = 25(x-2)^3$
$c(x-2)^3$

around $x=-3$
$f(x) = (-5)^3(x+3)^2 = -125(x+3)^2$

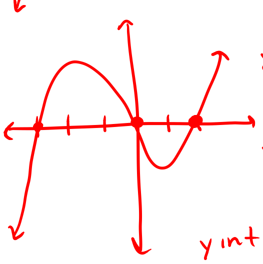
STEPS FOR GRAPHING POLYNOMIAL FUNCTIONS: 1. End behavior 2. Roots 3. Root behavior.

1) $f(x)=x(x-2)(x+3)$

1) $y=+x^3$

2) roots at $x=0, 2, -3$

3) near $x=0$, $f(x)=x(-2)(3)$
$f(x)=-6x$
$x=2$, $f(x)=2(x-2)(5)$
$f(x)=10(x-2)$
$x=-3$, $f(x)=-3(-5)(x+3)$
$f(x)=15(x+3)$



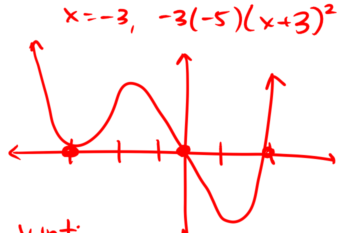
y int: $(0,0)$

2) $f(x)=x(x-2)(x+3)^2$

1) $y=+x^4$

2) $x=0, 2, -3$

3) near $x=0$, $(-2)(-3)^2=-18x$
$x=2$, $2(5)^2(x-2)=50(x-2)$
$x=-3$, $-3(-5)(x+3)^2=15(x+3)^2$



y int: $(0,0)$

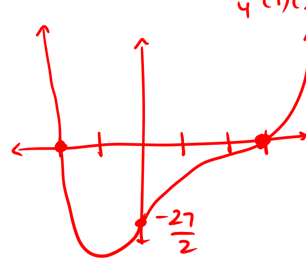
3) $f(x)=\frac{1}{4}(x+2)(x-3)^3$

1) $y=+x^4$

2) $x=-2, 3$

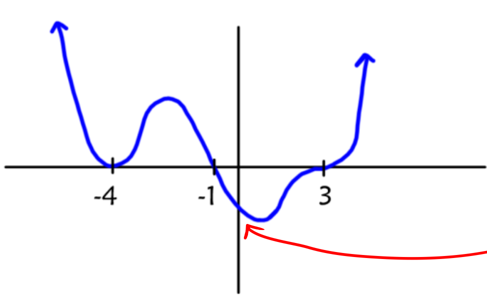
3) near $x=-2$
$\frac{1}{4}(x-2)(-5)^3 = -\frac{125}{4}(x-2)$

near $x=3$
$\frac{1}{4}(1)(x-3)^3$



y int: $\frac{1}{4}(2)(-3)^3$
$-\frac{27}{2}$
$(0, -\frac{27}{2})$

4) Write a possible equation for the function:



$y = +(x+4)^2(x+1)(x-3)^3$

If I plug in $x=0$,

$y=(4)^2(1)(-3)^3$

$y = -\#$

which is consistent w/ this graph