

# MORE RATIONAL FUNCTIONS

## GOING THROUGH THE HORIZONTAL ASYMPTOTE

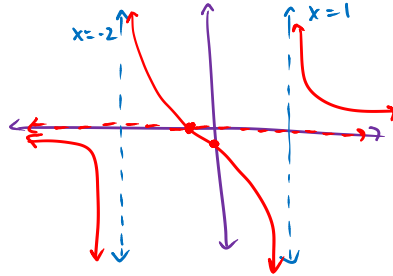
$$1) f(x) = \frac{3x+1}{x^2+x-2} = \frac{3x+1}{(x+2)(x-1)}$$

VA:  $x = -2, 1$

HA:  $y = 0$

x int:  $(-\frac{1}{3}, 0)$

y int:  $(0, -\frac{1}{2})$



**HORIZONTAL ASYMPTOTES:**  
Long run behavior of function

$$2) f(x) = \frac{(x-1)(x-4)}{(x-4)(x-3)(x+2)} = \frac{x-1}{(x-3)(x+2)}$$

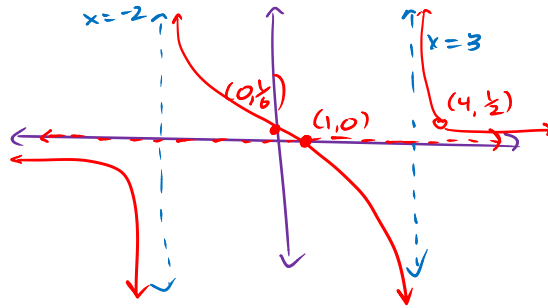
Hole:  $x = 4 \rightarrow (4, \frac{1}{2})$

VA:  $x = 3, -2$

HA:  $y = 0$

x int:  $(1, 0)$

y int:  $(0, \frac{1}{6})$



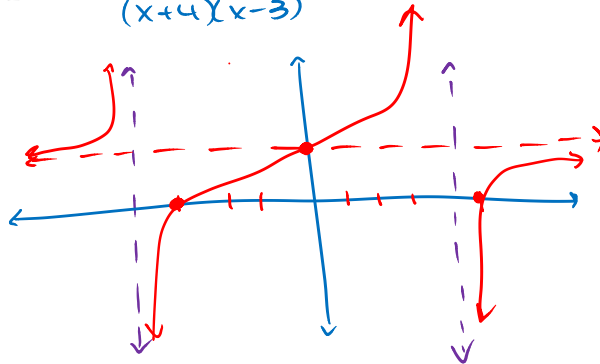
$$3) f(x) = \frac{2x^2 - 2x - 24}{x^2 + x - 12} = \frac{2(x-4)(x+3)}{(x+4)(x-3)}$$

VA:  $x = -4, 3$

HA:  $y = 2$

x int:  $(4, 0) (-3, 0)$

y int:  $(0, 2)$



Graph and show at what point the graph crosses the horizontal asymptote.

$$4) f(x) = \frac{x^2 + x - 2}{3x^2 + 10x - 8} = \frac{(x+2)(x-1)}{(3x-2)(x+4)}$$

VA:  $x = \frac{2}{3}$   $x = -4$

HA:  $y = \frac{1}{3}$

x int:  $(-2, 0)(1, 0)$

y int:  $(0, \frac{1}{4})$

Crossing the horiz. asym:

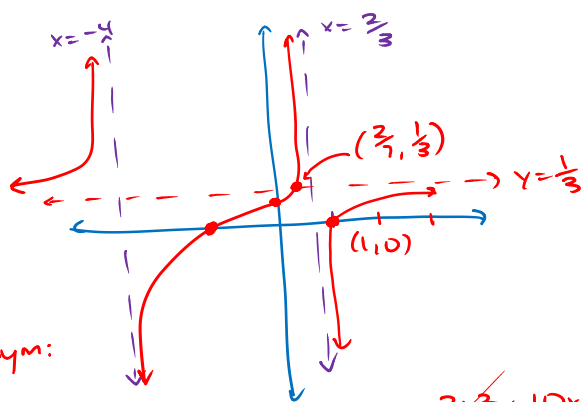
$$\frac{1}{3} = \frac{x^2 + x - 2}{3x^2 + 10x - 8}$$

$$3x^2 + 10x - 8 = 3(x^2 + x - 2)$$

~~$$3x^2 + 10x - 8 = 3x^2 + 3x - 6$$~~

~~$$7x = 2$$~~

~~$$x = \frac{2}{7}$$~~



TRY NUMBERS 5 - 8 ON YOUR OWN AND CHECK WITH THE KEY!  
I'LL GO OVER THESE WITH YOU TOMORROW IF YOU STILL HAVE QUESTIONS.

(5 AND 6 SHOULD CROSS THE HORIZONTAL ASYMPTOTE!)

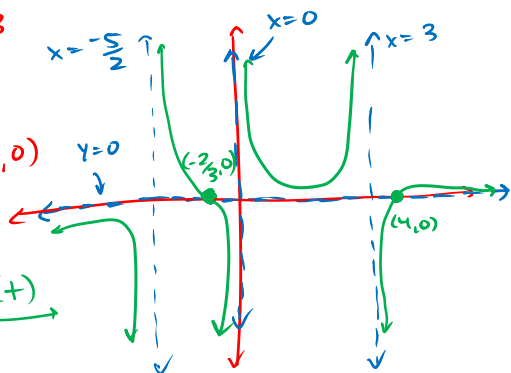
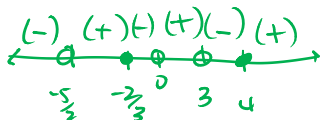
$$5) f(x) = \frac{3x^2 - 10x - 8}{2x^3 - x^2 - 15x} = \frac{(3x+2)(x-4)}{x(2x+5)(x-3)}$$

VA:  $x = 0, -\frac{5}{2}, 3$

HA:  $y = 0$

x int:  $(-\frac{2}{3}, 0)(4, 0)$

y int: none



$$6) f(x) = \frac{x^2 - 2x - 8}{2x^2 - 5x - 3} = \frac{(x-4)(x+2)}{(2x+1)(x-3)}$$

VA:  $x = -\frac{1}{2}$   $x = 3$

HA:  $y = \frac{1}{2}$

x int:  $(4, 0)(-2, 0)$

y int:  $(0, \frac{8}{3})$

CROSSES HORIZ:  $(13, \frac{1}{2})$

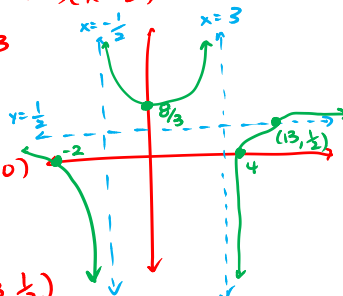
$$\frac{1}{2} = \frac{x^2 - 2x - 8}{2x^2 - 5x - 3}$$

~~$$2x^2 - 5x - 3 = 2(x^2 - 2x - 8)$$~~

~~$$2x^2 - 5x - 3 = 2x^2 - 4x - 16$$~~

~~$$-x = -13$$~~

~~$$x = 13$$~~



(REVIEW FROM YESTERDAY'S PROBLEMS)

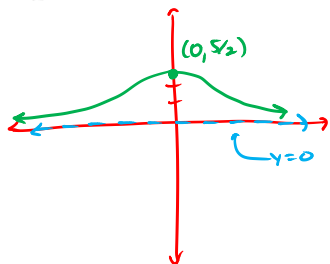
$$7) f(x) = \frac{5}{x^2 + 2}$$

VA: none

HA:  $y = 0$

x int: none

y int:  $(0, \frac{5}{2})$



$$8) f(x) = \frac{5x^2}{x^2 + 2}$$

VA: none

HA:  $y = 5$

x int:  $(0, 0)$

y int:  $(0, 0)$

