4.7
Notes
Day 3

EVEN MORE RATIONAL FUNCTIONS!

SLANT ASYMPTOTES: When the degree of the numerator is **exactly one more** than the degree of the denominator, the graph of the rational function will have a slant asymptote. To find the equation of the slant asymptote, perform long division by dividing the denominator into the numerator. As x gets very large, the remainder portion becomes very small, almost zero. So, to find the equation of the oblique asymptote, perform the long division and discard the remainder

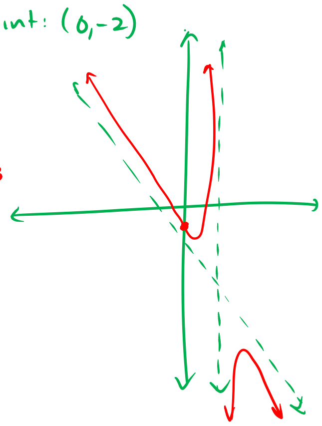
Graph the rational function.

1) $f(x)=\dfrac{-3x^2+2}{x-1}$

VA: $x=1$ x int: $(\pm\sqrt{2/3},0)$ y int: $(0,-2)$
HA: none!
slant: slant asym.

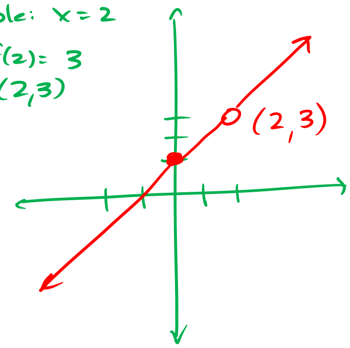
$$\begin{array}{r} -3x-3 \\ x-1\,\overline{)\,-3x^2+0x+2} \\ -(-3x^2+3x) \\ \hline -3x+2 \\ -(-3x+3) \\ \hline -1 \end{array}$$

$y=-3x-3$



2) $f(x)=\dfrac{x^2-x-2}{x-2}=\dfrac{(\cancel{x-2})(x+1)}{\cancel{x-2}}$

Hole: $x=2$
$f(2)=3$
$(2,3)$



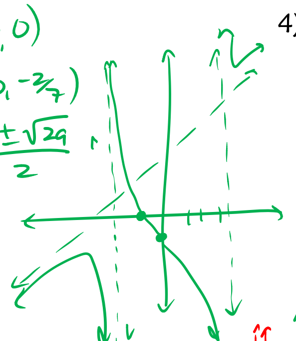
3) $f(x)=\dfrac{3x^3+2}{x^2-x-7}$

x int: $(\sqrt[3]{-2/3},0)$
y int: $(0,-2/7)$
VA: $x=\dfrac{1\pm\sqrt{29}}{2}$

slant: $\boxed{y=3x+3}$

$$\begin{array}{r} 3x+3 \\ x^2-x-7\,\overline{)\,3x^3+0x^2+0x+2} \\ -(3x^3-3x^2-21x) \\ \hline 3x^2+21x+2 \\ -(3x^2-3x-21) \\ \hline 24x+23 \end{array}$$

remainder



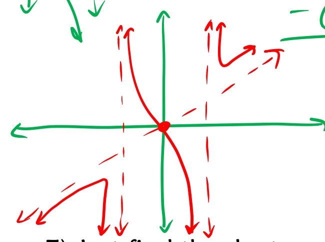
4) 3) $f(x)=\dfrac{x^3}{x^2-9}$

VA: $x=3$ $x=-3$
x int: $(0,0)$
y int: $(0,0)$

slant: $y=x$

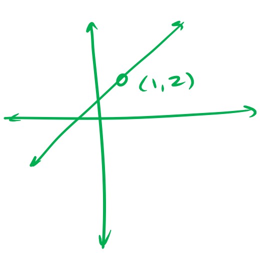
$$\begin{array}{r} x \\ x^2+0x-9\,\overline{)\,x^3+0x^2+0x+0} \\ -(x^3+0x^2-9x) \\ \hline 9x \end{array}$$

remainder



5) $f(x)=\dfrac{x^2-1}{x-1}=\dfrac{(x+1)(x-1)}{(x-1)}$

Hole: $(1,2)$

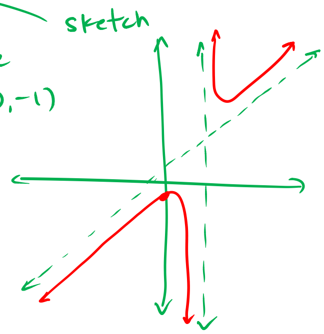


6) $f(x)=\dfrac{x^2+1}{x-1}$

sketch

VA: $x=1$ x int: none
slant: y int: $(0,-1)$
$y=x+1$

$$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2+0x+1} \\ -(x^2-x) \\ \hline x+1 \\ -(x-1) \\ \hline 2 \end{array}$$



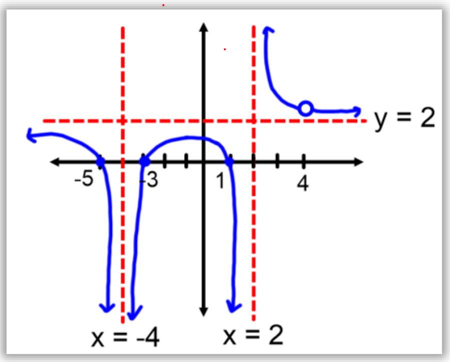
7) Just find the slant asymptote: $f(x)=\dfrac{3x^2+x+6}{3x+2}$

$$\begin{array}{r} x \\ 3x+2\,\overline{)\,3x^2+x+6} \\ -(3x^2+2x) \\ \hline -x+6 \end{array}$$

slant:
$y=x$

WRITING AN EQUATION FROM A GRAPH

Write a possible equation for the graph:

7) 

1) Hole at $x=4 \to \frac{x-4}{x-4}$

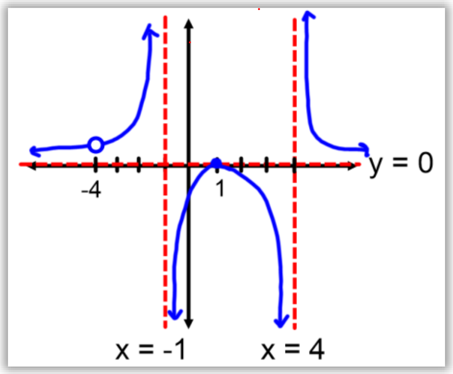
2) zeros at $x=-5,-3,1 \to (x+5)(x+3)(x-1)$ in numerator

3) VA at $x=-4$ & $x=-2 \to (x+4)(x-2)$ in denom.

4) $(x+4)$ should be even multiplicity

5) if HA. is at $y=2$, then degree must be the same & num. has to have a 2

Sooo... $$y=\frac{2(x+5)(x+3)(x-1)(x-4)}{(x+4)^2(x-2)(x-4)}$$

9) 

1) Hole at $x=-4 \Rightarrow \frac{(x+4)}{(x+4)}$

2) VA: $x=-1$, $x=4 \Rightarrow (x+1)(x-4)$ in denom.

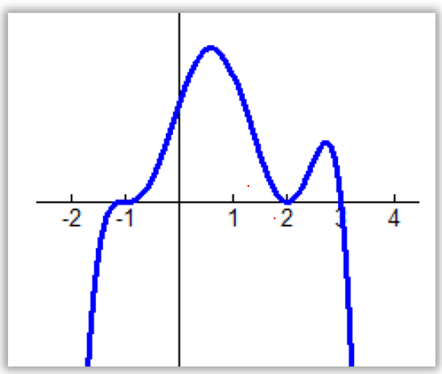
3) zero at $x=1 \Rightarrow (x-1)$ in num.

4) zero "bounces" back $\Rightarrow (x-1)$ has even multiplicity

5) HA: $y=0 \Rightarrow$ degree of denom. is bigger than num.

6) to keep consistent with (5), we need an odd degree for one vert. asymp.

Sooo.... $$y=\frac{(x-1)^2(x+4)}{(x+1)^3(x-4)(x+4)}$$ or $$y=\frac{(x-1)^2(x+4)}{(x+1)(x-4)^3(x+4)}$$

10) 

1) end behavior is even-negative

2) zero behaves like x^3 near -1 (goes through)

3) zero behaves like x^2 near 2 (bounces)

4) zero behaves like $-x$ near 3 (linear)

Soooo....

$$y=-(x+1)^3(x-2)^2(x-3)$$