

EXPONENTIAL FUNCTIONS

- Objectives: 1) Estimate a power of 2 in terms of a power of 10.
 2) Simplify expressions containing irrational exponents.
 3) Solve an exponential equation.
 4) Graph exponential functions using transformations.

ESTIMATING POWERS OF 2 USING POWERS OF 10

$$2^{10} \approx 10^3$$

1) 2^{40}

$$2^{40} = (2^{10})^4 \approx (10^3)^4 = 10^{12}$$

$$\approx 1,000,000,000,000$$

2) 2^{31}

$$2^{31} = (2^{10})^3 \cdot 2 \approx (10^3)^3 \cdot 2 = 2 \cdot 10^9$$

$$\approx 2 \times 1,000,000,000$$

$$\approx 2,000,000,000$$

SIMPLIFYING EXPRESSIONS WITH IRRATIONAL EXPONENTS

3) $(2^{1+\sqrt{3}})(2^{1-\sqrt{3}})$

$$2^{1+\sqrt{3} + 1-\sqrt{3}}$$

$$2^2$$

$$\boxed{4}$$

4) $(3^{\sqrt{2}})^{\sqrt{2}}$

$$3^2$$

$$\boxed{9}$$

5) $(3^{\sqrt{2}})^2$

$$(3^2)^{\sqrt{2}}$$

$$\boxed{9^{\sqrt{2}}}$$

SOLVING AN EXPONENTIAL EQUATION

Rewrite both sides of the equation so that you have the SAME BASE on both sides.

6) $10^{1-x} = 10^4$

$$1-x = 4$$

$$-x = 3$$

$$\boxed{x = -3}$$

7) $9^{x-1} = 27^2$

$$(3^2)^{x-1} = (3^3)^2$$

$$2x-2 = 6$$

$$2x = 8$$

$$\boxed{x = 4}$$

8) $x^{\frac{3}{4}} = 8$

$$(x^{\frac{3}{4}})^{-\frac{4}{3}} = (8)^{-\frac{4}{3}}$$

$$x = (2^3)^{-\frac{4}{3}}$$

$$x = 2^{-4}$$

$$\boxed{x = \frac{1}{16}}$$

9) $\frac{1}{3^{3x-2}} = 27$

$$3^{-(3x-2)} = 3^3$$

$$-3x+2 = 3$$

$$-3x = 1$$

$$\boxed{x = -\frac{1}{3}}$$

Recall:

$$\frac{1}{3} = 3^{-1}$$

$$\frac{1}{3^{-1}} = 3$$

10) $\left(\frac{1}{2}\right)^x = 8$

$$(2^{-1})^x = 2^3$$

$$-x = 3$$

$$\boxed{x = -3}$$

11) $3^{x^2-3x} = 81$

$$3^{x^2-3x} = 3^4$$

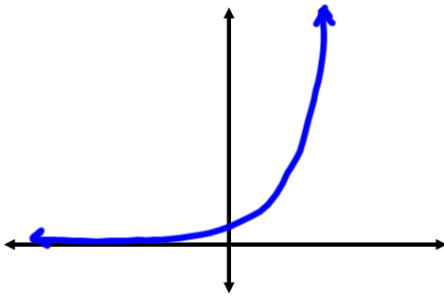
$$x^2-3x = 4$$

$$x^2-3x-4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x = 4, -1}$$

PROPERTIES OF AN EXPONENTIAL FUNCTION



$$y = b^x \quad \text{and } b > 1$$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

x-intercept: none

y-intercept: $(0, 1)$

VA: none

HA: $y = 0$

End Behavior: $x \rightarrow -\infty, y \rightarrow 0$

$x \rightarrow +\infty, y \rightarrow +\infty$

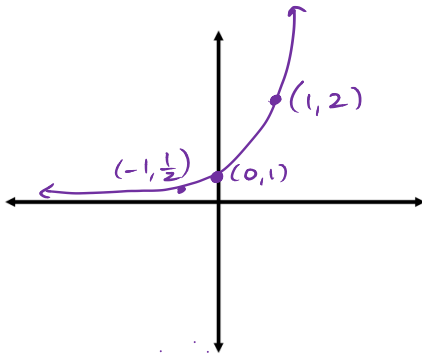
GRAPHING EXPONENTIAL FUNCTIONS USING TRANSFORMATIONS

Given $f(x)$, $-f(-x+a)+b$

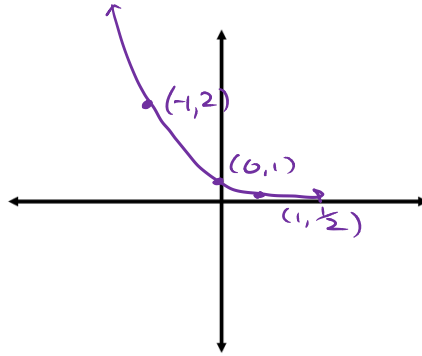
\swarrow F_x \swarrow F_y \swarrow $\frac{1}{R}$ shift
 $-(-x+a)$ $+b$ \swarrow $\frac{1}{b}$ shift

Sketch the graph of each exponential function.

12) $y = 2^x$

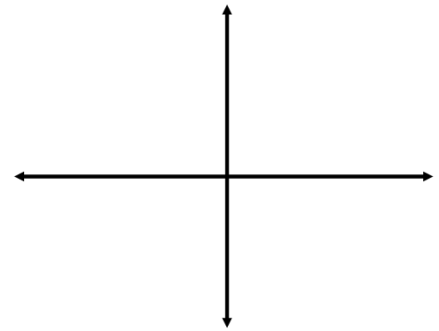


13) $y = 2^{-x}$

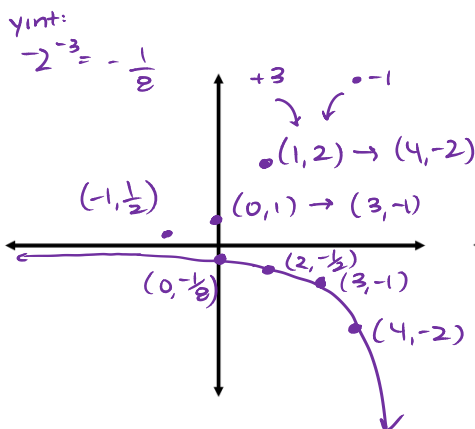


14) $y = \left(\frac{1}{2}\right)^x = 2^{-x}$

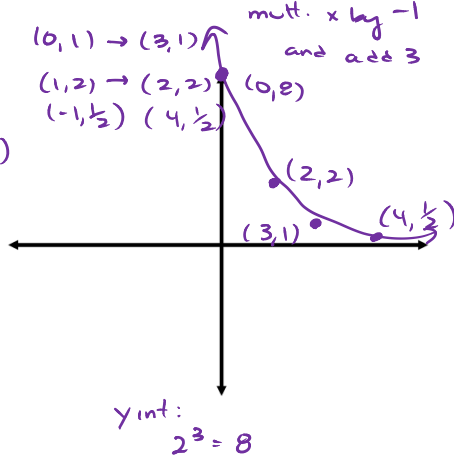
Same as #13!



15) $y = -2^{x-3}$



16) $y = 2^{-x+3} = 2^{-(x-3)}$



17) $y = 2^{x-4} - 1$

