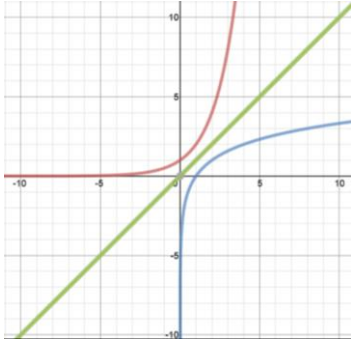


LOGARITHMIC FUNCTIONS

- Objectives: 1) Evaluate logarithms, including the natural log
 2) Solve an equation involving logarithms.
 3) Graph logarithmic functions (including the natural log) using transformations.

PROPERTIES OF A LOGARITHMIC FUNCTION



$$y = \log_b x \quad \text{and } b > 1$$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

y-intercept: none

VA: $x = 0$

HA: none

End Behavior: $x \rightarrow +\infty \Rightarrow y \rightarrow +\infty$
 $x \rightarrow 0 \Rightarrow y \rightarrow -\infty$

RECALL THE DEFINITION:

$$y = \log_b x \quad \text{if and only if} \quad b^y = x$$

Log form

Exponential form

A LOGARITHM IS THE INVERSE FUNCTION OF AN EXPONENTIAL FUNCTION.

Write the following in logarithmic form.

1) $n = 4^x$

$$\log_4 n = x$$

2) $3^4 = 81$

$$\log_3 81 = 4$$

3) $2^{-3} = \frac{1}{8}$

$$\log_2 \frac{1}{8} = -3$$

Write the following in exponential form.

4) $\log_5 125 = 3$

$$5^3 = 125$$

5) $\log_n t = y$

$$n^y = t$$

6) $\log_3 3^x = 5$

$$3^5 = 3^x$$

EVALUATING LOGS – USE THE DEFINITION OF A LOG TO REWRITE AND EVALUATE

7) $\log_{36} \frac{1}{\sqrt{6}} = x$

$$36^x = \frac{1}{\sqrt{6}}$$

$$(6^2)^x = \frac{1}{6^{\frac{1}{2}}}$$

$$6^{2x} = 6^{-\frac{1}{2}}$$

$$\boxed{x = -\frac{1}{4}}$$

8) $\left(\log_9 \frac{1}{3}\right)^2$

$$\log_9 \frac{1}{3} = x$$

$$9^x = \frac{1}{3}$$

$$(3^2)^x = 3^{-1}$$

$$x = -\frac{1}{2}$$

$$\left(-\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}}$$

9) $-3(\log \sqrt[3]{10})$

$$\log_{10} \sqrt[3]{10} = x$$

$$10^x = 10^{\frac{1}{3}}$$

$$x = \frac{1}{3}$$

$$-3\left(\frac{1}{3}\right)$$

$$\boxed{-1}$$

10) $\ln e^3$

$$\log_e e^3 = x$$

$$e^x = e^3$$

$$\boxed{3}$$

SOLVING A LOG EQUATION

11) $\log_4(x-5) = 3$

$$4^3 = x-5$$

$$4^3 + 5 = x$$

$$\boxed{69 = x}$$

12) $\ln e^{3x-5} = 10$

$$e^{10} = e^{3x-5}$$

$$3x-5 = 10$$

$$\boxed{x=5}$$

13) $e^{3x-5} = 10$

$$\ln e^{3x-5} = \ln 10$$

$$3x-5 = \ln 10$$

$$3x = \ln 10 + 5$$

$$\boxed{x = \frac{\ln 10 + 5}{3}}$$

14) $\log_3(3x-2) = 4$

$$3^4 = 3x-2$$

$$81 = 3x-2$$

$$83 = 3x$$

$$\boxed{x = \frac{83}{3}}$$

15) $\log_4 4^{2x-5} = 7$

$$4^7 = 4^{2x-5}$$

$$7 = 2x-5$$

$$\boxed{x=6}$$

16) $e^{2x+2} = 5$

$$\ln e^{2x+2} = \ln 5$$

$$2x+2 = \ln 5$$

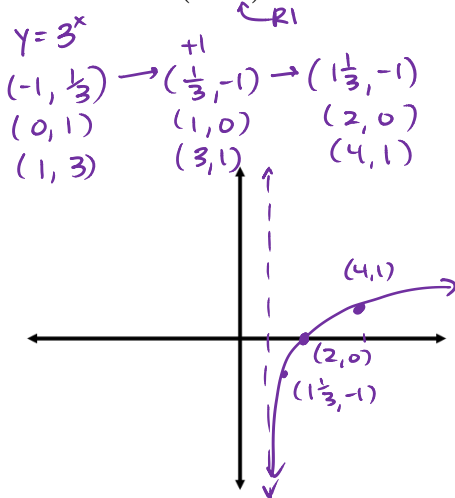
$$2x = \ln 5 - 2$$

$$\boxed{x = \frac{\ln 5 - 2}{2}}$$

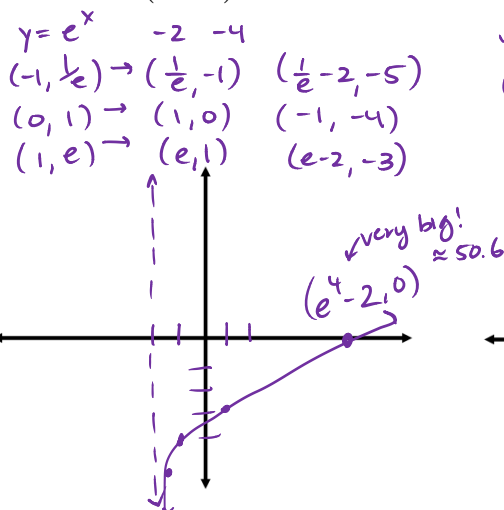
THE GRAPH OF A LOGARITHM

Sketch the graph of the logarithmic function.

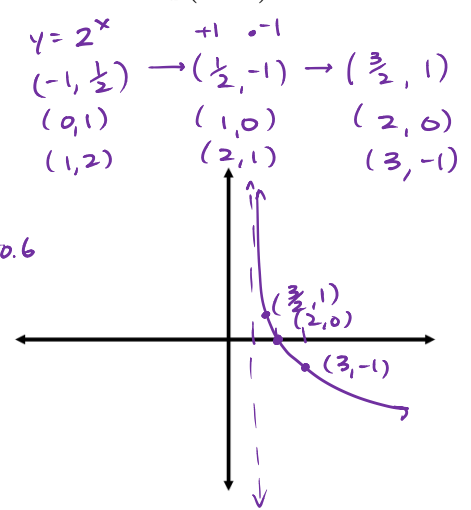
12) $y = \log_3(x-1)$



13) $y = \ln(x+2) - 4$



14) $y = -\log_2(x-1)$



Find y intercept:
let $x=0$

$$y = \ln(x+2) - 4$$

$$y = \ln(0+2) - 4$$

$$y = \ln(2) - 4$$

Find x intercept:
let $y=0$

$$0 = \ln(x+2) - 4$$

$$4 = \ln(x+2)$$

$$e^4 = x+2$$

$$\boxed{e^4 - 2 = x}$$