# PROPERTIES OF LOGARITHMS

Objectives: 1) Condense and expand expressions using log properties
2) Find an x intercept using log properties
3) Use a logarithm to solve an equation
4) Use change of base to simplify expressions

## THREE GOLDEN RULES!

**PROPERTIES OF LOGS:**

1) **Log of a Product:** $\log_x ab = \log_x a + \log_x b$

2) **Log of a Quotient:** $\log_x \frac{a}{b} = \log_x a - \log_x b$

3) **Log of a Power:** $\log_x a^b = b\log_x a$

**EXPONENTS:**

1) **Product of Powers:** $x^a \bullet x^b = x^{a+b}$

2) **Quotient of Powers:** $\frac{x^a}{x^b} = x^{a-b}$

3) **Power of a Power:** $\left(x^a\right)^b = x^{ab}$

## EVALUATE!

Given $\log_b 2 = A$, $\log_b 5 = B$, and $\log_b 7 = C$, evaluate the following:

1) $\log_b \sqrt[5]{.4} = \log_b \left(\frac{2}{5}\right)^{1/5}$

$\frac{1}{5}\log_b \frac{2}{5}$

$\frac{1}{5}(\log_b 2 - \log_b 5)$

$\frac{1}{5}(A - B)$

2) $\log_b \left(\frac{7}{16}\right)$

$\log_b 7 - \log_b 16 \quad 16 = 2^4$

$\log_b 7 - 4\log_b 2$

$C - 4(A)$

3) $\log_b 3.5$

$\log_b \frac{7}{2}$

$\log_b 7 - \log_b 2$

$C - A$

## EXPAND!

**Rewrite as sums and differences so that no products, quotients or powers appear.**

4) $\log_a \left(x\sqrt{x^2+1}\right) \quad x > 0$

$\log_a x + \log_a \sqrt{x^2+1}$

$\log_a x + \frac{1}{2}\log_a(x^2+1)$

STOP! Do NOT split this up!

5) $\log_a \frac{\sqrt{x^2+1}}{x^3(x+1)^4} \quad x > 0$

$\log_a \sqrt{x^2+1} - \log_a x^3 - \log_a (x+1)^4$

$\frac{1}{2}\log_a(x^2+1) - 3\log_a x - 4\log_a(x+1)$

## CONDENSE!

**Simplify into a single logarithm with coefficient 1.**

6) $\log_a 7 + 4\log_a 3$

$\boxed{\log_a 7 \cdot 3^4}$

7) $\frac{1}{3}\left[\log_2 x + \log_2 (x+1)\right]$

$\frac{1}{3}\left[\log_2(x(x+1))\right]$

$\boxed{\log_2 \sqrt[3]{x(x+1)}}$

8) $\log_a x + \log_a 9 + \log_a \left(x^2+1\right) - \log_a 5$

$\boxed{\log_a \frac{9x(x^2+1)}{5}}$

## YOU TRY

Expand:

9) $\log_3 \dfrac{x^2y}{\sqrt[5]{3x-1}}$

$2\log_3 x + \log_3 y - \frac{1}{5}\log_3(3x-1)$

10) $\log_4 5x^3y$

$\log_4 5 + 3\log_4 x + \log_4 y$

11) $\log_7 h^2j^{11}k^{-5}$

$2\log_7 h + 11\log_7 j - 5\log_7 k$

12) $\ln\dfrac{\sqrt{3x-5}}{7}$

$\frac{1}{2}\ln(3x-5) - \ln 7$

Condense:

13) $\dfrac{1}{2}\log x + 3\log(x+1)$

$\log(\sqrt{x}\cdot(x+1)^3)$

14) $2\ln(x+2) - \ln x$

$\ln\dfrac{(x+2)^2}{x}$

15) $2\left[\log_3 x + 3\log_3(x-2)\right]$

$2(\log_3 x\cdot(x-2)^3)$

$2\log_3(x(x-2)^3)$

$\log_3(x(x-2)^3)^2$

$\boxed{\log_3(x^2(x-2)^6)}$

## USING LOGS TO SOLVE EXPONENTIAL EQUATIONS

Sometimes, when solving an exponential equation, we must use logs. There are two types of exponential equations – those that can be written with the same base, and those that cannot.

**SAME BASE VS. NOT SAME BASE**

$4^x = 32$ | $4^x = 31$

$(2^2)^x = 2^5$

Whenever you get "stuck" in an exponential or log equation, rewrite it as its opposite!
***YOU CAN SOLVE IT IN ONE OF TWO WAYS.***

Solve the following exponential equations.

1) $4 \cdot 10^x = 1$

$10^x = \frac{1}{4}$

$\boxed{\log_{10}\frac{1}{4} = x}$

2) $2^{-3x} = 45$

$\log_2 45 = -3x$

$\boxed{\dfrac{\log_2 45}{-3} = x}$

When solving for an x int!

3) $y = 3^x - 5$

$0 = 3^x - 5$

$5 = 3^x$

$\boxed{\log_3 5 = x}$

Another way to solve: **The inverse of an exponential function is a log!**

7) $8^x = 23$

$\log_8 8^x = \log_8 23$

$\boxed{x = \log_8 23}$

8) $4^x = 7$

$\log_4 4^x = \log_4 7$

$\boxed{x = \log_4 7}$

**CHANGE OF BASE:**

$\log_a x = \dfrac{\log_b x}{\log_b a}$

EX) $\log_8 23 = \dfrac{\log 23}{\log 8}$