EQUATIONS AND INEQUALITIES W/ LOGS AND EXPONENTS

Objectives: 1) Solve log equations, with respect to a restricted domain.
2) Solve log inequalities with respect to a restricted domain.
3) Use inverses to solve a log or exponential equation or inequality.

SOLVING LOG/EXPONENTIAL EQUATIONS: USE INVERSE OPERATIONS!

1) $\log_2(y+2) - 1 = \log_2(y-2)$

$\log_2(y+2) - \log_2(y-2) = 1$

$\log_2 \frac{(y+2)}{(y-2)} = 1$

$\frac{y+2}{y-2} = 2^1$

$y+2 = 2(y-2)$

$y+2 = 2y-4$

$\boxed{y=6}$

2) $\log_{10}(\log_{10} x) = 4$

$10 \qquad 10$

$\log_{10} x = 10^4$

$\boxed{10^{10^4} = x}$

3) $3^{2x-1} = 6^x$

$\ln 3^{2x-1} = \ln 6^x$

$(2x-1)\ln 3 = x \ln 6$

$2x(\ln 3) - \ln 3 = x \ln 6$

$2x\ln 3 - x\ln 6 = \ln 3$

$x(2\ln 3 - \ln 6) = \ln 3$

$\boxed{x = \frac{\ln 3}{2\ln 3 - \ln 6}}$

Find the intercepts for

4) $y = 4(5^{2x}) - 5^x - 5$

y int: $x=0$

$y = 4(5^0) - 5^0 - 5 = -2 \quad (0,-2)$

x int: $y=0$ let $t = 5^x$

$0 = 4t^2 - t - 5$

$0 = (4t-5)(t+1)$

$t = \frac{5}{4} \qquad t = -1$

$5^x = \frac{5}{4} \qquad 5^x = -1$

$\boxed{\log_5 \frac{5}{4} = x}$ no real sol!

Extraneous solutions!

5) $2\log_5(-x) - \frac{3}{2}\log_5 25 = -5$

$\log_5 \frac{(-x)^2}{(25)^{3/2}} = -5$

$\frac{x^2}{(5^2)^{3/2}} = 5^{-5}$

$\frac{x^2}{5^3} = \frac{1}{5^5}$

$x^2 = \frac{1}{5^5} \cdot 5^3 = \frac{1}{5^2}$

$x = \pm\sqrt{\frac{1}{25}} \Longrightarrow x = \pm \frac{1}{5}$

Check w/ original:

$x = \frac{1}{5}$ gives a neg. arg.

So only solution is:

$\boxed{x = \frac{-1}{5}}$

YOU TRY!

6) $e^{\ln x} = 2$

$\boxed{x=2}$

7) $\log_{10}[\log_2(\log_7 x)] = 0$

$10^0 = \log_2(\log_7 x))$

$2^1 = \log_7 x$

$7^2 = x$

$\boxed{x=49}$

8) $10^{\log_{10} x} = -2$

$x = -2$

$\boxed{\text{no solution!}}$

YOU TRY!

9) $\log_{10}\left(x^2 - 21x\right) = 2$

$10^2 = x^2 - 21x$

$x^2 - 21x - 100 = 0$

$(x-25)(x+4) = 0$

$\boxed{x = 25, -4}$

Both sol. work

10) $\log_2 x + \log_2(22 - 5x) = 3$

$\log_2 x(22-5x) = 3$

$2^3 = 22x - 5x^2$

$0 = 5x^2 - 22x + 8$

$0 = 5x^2 - 20x - 2x + 8$

$0 = 5x(x-4) - 2(x-4)$

$0 = (5x-2)(x-4)$

$\boxed{x = \frac{2}{5} \quad x = 4}$

Both solutions work!

SOLVING LOG/EXPONENTIAL INEQUALITIES:

1. Logging or exponentiating both sides of an equation **does NOT change** the inequality direction.

2. Log$_b$ a is negative when $0 < a < 1$; multiplication/division **DOES change** the inequality direction.

3. Logs have domain restrictions. These restrictions must be considered in inequality situations.

11) $4\left(.1^x - 5\right) < 8$

$\frac{1}{10}^x - 5 < 2$

$\frac{1}{10}^x < 7$

$\log_{\frac{1}{10}} \frac{1}{10}^x < \log_{\frac{1}{10}} 7$

$x < \log_{\frac{1}{10}} 7$

12) Domain: $4 - 5x > 0 \quad x < \frac{4}{5}$

$e^{\ln(4-5x)} \le e^2$

$4 - 5x \le e^2$

$-5x \le e^2 - 4$

$x \ge \frac{e^2 - 4}{-5}$

$\boxed{\left[\frac{e^2-4}{-5}, \frac{4}{5}\right)}$

13) Domain: $\mathbb{R}$,

$e^{4-5x} \le 2$

$\ln e^{4-5x} \le \ln 2$

$4 - 5x \le \ln 2$

$-5x \le \ln 2 - 4$

$x \ge \frac{\ln 2 - 4}{-5}$

$\boxed{\left[\frac{\ln 2 - 4}{-5}, \infty\right)}$

14) $\ln(x-5) + \ln(x) \le \ln 36$

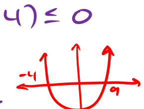
$\ln(x-5)(x) \le \ln 36$

$x^2 - 5x \le 36$

$x^2 - 5x - 36 \le 0$

$(x-9)(x+4) \le 0$

test: $+ \quad - \quad +$ (-4, 9)



(or sketch)

$\boxed{[-4, 9]}$

But original domain $x > 5$

$[5, 9]$

15) $e^{-x^2} < e^{-24}$

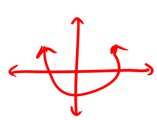
$-x^2 < -24$

$x^2 > 24$

$x^2 - 24 > 0$

$x^2 = 24$

$x = \pm 2\sqrt{6}$



$(-\infty, 2\sqrt{6}) \cup (2\sqrt{6}, \infty)$

16) $\ln \frac{3x-2}{4x+1} > \ln 2$

D: $(-\infty, -\frac{1}{4})(\frac{2}{3}, \infty)$

$\frac{3x-2}{4x+1} > 2$

$\frac{3x-2}{4x+1} - 2 > 0$

$\frac{3x-2-2(4x+1)}{4x+1} > 0$

$\frac{-5x-4}{4x+1} > 0$

$\frac{5x+4}{4x+1} < 0$

$+ \quad - \quad +$ ($-\frac{4}{5}$, $-\frac{1}{4}$)

$\boxed{\left(-\frac{4}{5}, -\frac{1}{4}\right)}$