COMPOUND INTEREST

Objectives: 1) Find the amount of time it takes to double an investment.
2) Compute the effective rate when compounded at different periods of time.
3) Compute the nominal rate when given the effective rate.

FORMULAS

SIMPLE INTEREST (ANNUAL)	COMPOUND INTEREST (N TIMES PER YEAR)	COMPOUND INTEREST (CONTINUOUS)	DOUBLING TIME
$A = P(1+r)^t$	$A = P\left(1+\frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$	$T_2 = \frac{\ln 2}{r}$

VARIABLES:
A=amount in account
r=annual interest rate
n=number of times per year
P=principal (invested)
t=time in years
T_2 = doubling time

VOCABULARY:
NOMINAL RATE: The rate in name only. Meaning, the nominal rate does not take into account how interest is compounded!
PERIODIC RATE: The interest rate being paid at each time period.
EFFECTIVE RATE (APR): The simple interest rate that gives the same result as the nominal rate compounded.

Examples
1. Compare the account totals for $12,000 deposited into quarterly and continuously compounded accounts after 5 years with annual interest rate of 9%. Also compare the nominal and effective rates.

Quarterly
$A = 12{,}000\left(1+\frac{.09}{4}\right)^{4\cdot 5} = 18726.11$
$\boxed{\$18{,}726.11}$

nominal rate: 9%
effective rate: t=1
$\left(1+\frac{.09}{4}\right)^4 \approx 1.09308$
$\boxed{9.3\% \text{ effective rate}}$

Continuously:
$A = Pe^{rt}$
$A = 12{,}000e^{.09(5)} = 18{,}819.746$
$\boxed{\$18{,}819.74}$

nominal rate: 9%
effective rate: t=1
$e^{.09(1)} \approx 1.09417$
$\boxed{9.4\% \text{ effective rate}}$

2. Find the doubling time for the continuous account in example 1.

$24000 = 12000e^{.09t}$
$2 = e^{.09t}$
$\ln 2 = .09t$
$t = \frac{\ln 2}{.09} \approx 7.7$ About 7.7yrs

Alternatively:
(we are just approximating)
$\ln 2 \approx .69$
$\frac{.69}{.09} = \frac{69}{100}\cdot\frac{100}{9} \approx \frac{72}{9}$
$\boxed{8 \text{ yrs}}$

3. You need $15,000 for a down payment on a new home in 5 years. If the bank guarantees 4% APR on a certificate of deposit (CD) that compounds monthly, how much needs to be deposited into the account today? What if the interest is compounded continuously?

$$15000 = P\left(1+\frac{.04}{12}\right)^{12(5)}$$

$$\frac{15000}{\left(1+\frac{.04}{12}\right)^{60}} = P \qquad \boxed{P = \$12,285.05}$$

Round up here b/c you at least need this much money

$$15000 = Pe^{.04(5)}$$

$$\frac{15000}{e^{.2}} = P \qquad \boxed{P = \$12,280.97}$$

4. What interest rate (compounded annually) is needed for a $4000 deposit to grow to $5000 in 3 years?

$$5000 = 4000\left(1+\frac{r}{1}\right)^3$$

$$\frac{5}{4} = (1+r)^3$$

$$\sqrt[3]{\frac{5}{4}} = 1+r$$

$$\sqrt[3]{\frac{5}{4}} - 1 = r \qquad \boxed{7.7\%}$$

$$.077 = r$$

5. a) Given a nominal rate of 6% per annum compounded continuously, compute the effective rate.
   b) Given an effective rate of 6% per annum, compute the nominal rate compounded continuously.

a) Using $A = Pe^{rt}$

$r = .06 \quad t = 1 \quad P = \text{anything}$

effective rate is after 1 year

$$A = Pe^{.06}$$

$$\approx P(1.06183)$$

effective rate!

$$\boxed{\approx 6.18\%}$$

b) using $A = Pe^{rt}$ $\quad A = P(1.06) \quad t = 1$

$$P(1.06) = Pe^{r \cdot 1}$$

$$1.06 = e^r$$

$$\ln 1.06 = \ln e^r$$

nominal rate

$$\ln 1.06 = r$$

$$r \approx .05826$$

$$\boxed{\approx 5.83\%}$$