(CONTINUOUS) EXPONENTIAL GROWTH AND DECAY

Objectives: 1) Determine the growth constant of a population
2) Find the doubling time for a population
3) Determine the decay constant when given the half life

(CONTINUOUS) EXPONENTIAL GROWTH FUNCTIONS (GENERAL FORM)

$$y = ae^{bx}$$

a and b are positive constants

(CONTINUOUS) EXPONENTIAL DECAY FUNCTIONS (GENERAL FORM)

$$y = ae^{bx}$$

a is positive and b is negative

POPULATION GROWTH: $N(t) = N_0 e^{kt}$

k = growth constant $\quad N_0$ = the size of the population at t = 0

1) In 2000, the nations of Mali and Cuba had similar size populations: Mali 11.2 million, Cuba 11.1 million. However, the relative growth rate for Mali was 3.1%/year, whereas that for Cuba was 0.7%/year.
a) Assuming continuous exponential growth at the given rates, make projections for each population in the year 2015.
b) When might the population of Mali reach 20 million? What would the population of Cuba be at the same time?

a) $t=15 \quad k=.031$

$N = 11.2e^{.031(15)}$

$N = \boxed{17.83 \text{ million}}$

$t=15 \quad k=.007$

$N = 11.1e^{.007(15)}$

$\boxed{N = 12.33 \text{ million}}$

b) $20 = 11.2e^{.031t}$

$\frac{20}{11.2} = e^{.031t}$

$\ln\frac{20}{11.2} = .031t$

$\frac{\ln\frac{20}{11.2}}{.031} = t \quad \boxed{t \approx 18.7 \text{ yrs}}$

STO!

Cuba:

$N = 11.1e^{.007t}$ use stored value!

$\boxed{N = 12.65 \text{ million}}$

2) At the start of an experiment in a biology lab, 1500 bacteria are present in a colony. Two hours later, the size of the population is found to be 1750. Assume that the population size grows exponentially.
   a) How many bacteria were there 1.5 hours after the experiment began?
   b) When will the population reach 5200?
   c) How long does it take for the population to double?

OFTEN TWO STEPS ARE NEEDED:
1) Use given info to find *k*.
2) Answer the question.

$N=1750 \quad N=1500 \quad t=2$

a) $1750 = 1500e^{k \cdot 2}$

$\frac{1750}{1500} = e^{2k}$

$\ln\frac{1750}{1500} = 2k$

$\boxed{k = \frac{\ln\frac{1750}{1500}}{2}}$ STO!

$(k \approx .077)$

b) $5200 = 1500e^{kt}$ use stored k!

$\frac{5200}{1500} = e^{kt}$

$\ln\frac{5200}{1500} = kt$

$\boxed{t = \frac{\ln\frac{5200}{1500}}{k}}$

$t \approx 16.13$

c) $\left(T_2 = \frac{\ln 2}{k}\right)$ use stored k!

$3000 = 1500e^{kt}$

$2 = e^{kt}$

$\ln 2 = kt \quad t \approx 8.99 \text{ yrs}$

$\boxed{\frac{\ln 2}{k} = t} \Rightarrow \boxed{\approx 9 \text{ yrs}}$

HALF-LIFE: The half-life of a radioactive substance is the time required for half of a given sample to disintegrate. The half-life is an intrinsic property of the substance; it does not depend on the given sample size.

$$N(t) = N_0 e^{kt}$$

k = decay constant (will be negative!) N_0 = the size of the population at t = 0

3 The half-life of radium-226 is 1620 years.

a. How much of an initial 2-g sample remains after 5 years?

b. Find the time required for 80% of the 2-g sample to decay.

a) Find k first!

$\frac{1}{2} N_0 = N_0 e^{kt}$

$\frac{1}{2} = e^{k \cdot 1620}$

$\ln \frac{1}{2} = 1620k$

$\frac{\ln \frac{1}{2}}{1620} = k$ STO!

$(k \approx -4.23)$

2 grams → 5 yrs

$N = 2e^{k \cdot 5}$ use stored value!

$\boxed{N = 1.995 \text{ grams}}$

Basically nothing decayed even after 5 yrs! Sick!

b) 80% decays ⇒ 20% left!

$N = .20 N_0$

$N = \frac{1}{5} N_0$

$\frac{1}{5} \cancel{N_0} = \cancel{N_0} e^{kt}$ use stored value!

N_0 will cancel:

$\frac{1}{5} = e^{kt}$

$\ln \frac{1}{5} = kt$

$\boxed{\frac{\ln \frac{1}{5}}{k} = t \quad t \approx 3761.5}$

It takes approximately 3,762 yrs for 80% of a 2gram sample to decay.

AAAH!!! Crazy!!