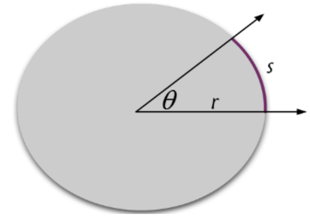
RADIAN MEASURES

OBJECTIVES: 1) Calculate the radian measure of an angle.
2) Find the arc length of a circle and find the area of a sector of a circle.
3) Find the angular and linear speed of an object.

THE RADIAN MEASURE OF AN ANGLE:

Place the vertex of the angle at the center of a circle of radius r.
Let s denote the length of the arc intercpeted by the angle.
The radian measure θ is the ratio of the arc length to the radius.

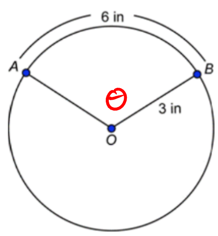
$$\theta = \frac{s}{r}$$



The lengths s and r have units that cancel out so θ is *dimensionless* but has title radians.

1) Determine the radian measure for the angle.

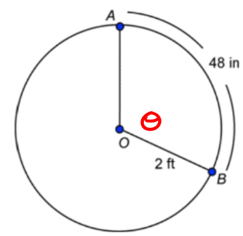
a)



$\theta = \frac{6}{3} = 2$

$\boxed{2 \text{ radians}}$

b)

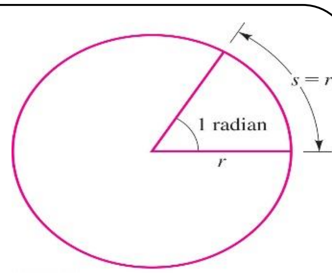


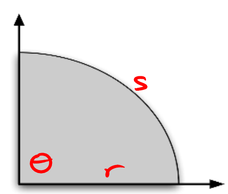
$\theta = \frac{48\text{ in}}{2\text{ ft}} = \frac{48\text{ in}}{24\text{ in}}$

$= \boxed{2 \text{ radians}}$

RADIAN MEASURE:

In a circle, 1 radian is the measure of the central angle that interecepts an arc equal in length to the radius of the circle.





Quarter Circle:

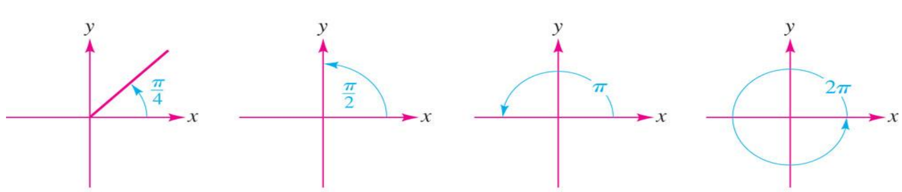
$s = \frac{1}{4} \cdot 2\pi r = \frac{1}{2}\pi r$

$\theta = \frac{s}{r} = \frac{\frac{1}{2}\pi r}{r} = \frac{1}{2}\pi = \frac{\pi}{2}$

$\frac{\pi}{2}$ radians $= 90°$

$\frac{\pi}{180}$ radians $= 1°$

(a) 1 radian (b) 2 radians (c) 3 radians



2) Convert from radians to degrees or vice versa.

a) 120°

$120 \cdot \frac{\pi}{180} = \boxed{\frac{2\pi}{3}}$

b) $\frac{\pi}{6}$

$\frac{\pi}{6} \cdot \frac{180}{\pi} = \boxed{30°}$

CONVERTING MEASURES:

$1° = \frac{\pi}{180}$ radians

1 radian $= \frac{180}{\pi}$

ARC LENGTH AND AREA OF SECTORS

ARC LENGTH: $s = r\theta$

3) Find the arc length on a circle with radius 4in and angle 45°.

$45° = 45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ radians

$s = r\theta = 4 \cdot \frac{\pi}{4} = \boxed{\pi \text{ in}}$

AREA OF SECTOR: $A = \frac{1}{2}r^2\theta$

4) Find the area of a sector when $r = \sqrt{3}$ and $\theta = 30°$.

$A = \frac{1}{2}r^2\theta$

$30° = 30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$

$A = \frac{1}{2} \cdot 3 \cdot \frac{\pi}{6} = \boxed{\frac{\pi}{4} \text{ u}^2}$

5) A sector has an area of 100 cm² and a central angle of $\frac{1}{2}$ radians. Find the radius and the arc length.

$100 = \frac{1}{2}r^2\theta$

$\theta = \frac{s}{r} = .5$

$100 = \frac{1}{2}r^2 \cdot \frac{1}{2}$

$100 = \frac{1}{4}r^2$

$400 = r^2$

$r = \pm 20$

$\boxed{r = 20 \text{ cm}}$

$\theta = \frac{s}{r}$ $\quad s = \theta r$

$s = \frac{1}{2} \cdot 20$

$\boxed{s = 10 \text{ cm}}$

APPLICATION: SPEEDS ON A WHEEL

ANGULAR SPEED: $\omega = \frac{\Delta\theta}{\Delta t}$ Units: $\frac{\text{radians}}{\text{time}}$

LINEAR SPEED: $v = \frac{\Delta d}{\Delta t}$ Units: $\frac{\text{in, cm, ft...}}{\text{time}}$

linear measurement

6) A CD rotates at 180 rpm (revolutions per minute). Calculate the angular speed and the linear speed for a point 6 cm from the center.

$\omega = \frac{180 \text{ revol.}}{\text{minute}} \cdot \frac{2\pi \text{ radians}}{\text{revol.}} = \boxed{\frac{360\pi \text{ radians}}{\text{min}}}$

$v = r\omega$ $\quad v = \frac{360\pi}{\text{min}} \cdot 6 \text{ cm} = \boxed{\frac{2160\pi \text{ cm}}{\text{min}}}$

EQUATION RELATING BOTH:

$v = r\omega$ or $\omega = \frac{v}{r}$

7) A belt connects two spinning discs. If the radius of the larger is 15cm, th e radius of the smaller is 5cm and the angular velocity of the larger is 3000rpm, find the angular velocity of the smaller.

$\omega_{big} = \frac{3000 \text{ rev.}}{\text{min}} \cdot 2\pi = \frac{6000\pi \text{ radians}}{\text{min}}$

$v_{big} = \frac{6000\pi}{\text{min}} \cdot 15 \text{cm} = \frac{90{,}000\pi \text{ cm}}{\text{min}}$

$\omega_{small} = \frac{v}{r}$

$\omega_{small} = \frac{\frac{90{,}000\pi \text{ cm}}{\text{min}}}{5 \text{ cm}} = \boxed{\frac{18{,}000\pi}{\text{min}}}$