

TRIGONOMETRIC FUNCTIONS OF ANGLES

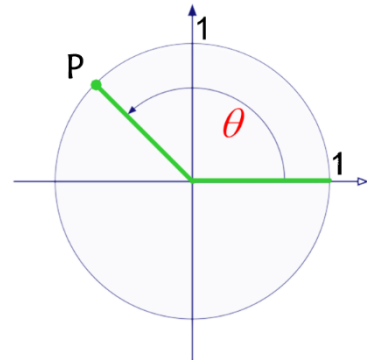
- OBJECTIVES:**
- 1) Use the unit circle to calculate trig functions of an angle.
 - 2) Verify that a point is on the unit circle.
 - 3) Use a calculator to evaluate trig functions.

TRIGONOMETRIC FUNCTIONS:

The functions $\cos(\)$ and $\sin(\)$ produce the x and y coordinates on a unit circle for a given angle.

$$\cos(\theta) = x \quad \sin(\theta) = y$$

$$P(x, y) = P(\cos(\theta), \sin(\theta))$$



OTHER DEFINITIONS:

Name	Abbreviation
Cosine	cos
Sine	sin
Tangent	tan
Secant	sec
Cosecant	csc
Cotangent	cot

$$\cos(\theta) = x$$

$$\sin(\theta) = y$$

$$\tan(\theta) = \frac{y}{x} \quad (x \neq 0)$$

$$\sec(\theta) = \frac{1}{x} \quad (x \neq 0)$$

$$\csc(\theta) = \frac{1}{y} \quad (y \neq 0)$$

$$\cot(\theta) = \frac{x}{y} \quad (y \neq 0)$$

EXAMPLES:

1) Use the unit circle to find:

a) $\cos\left(\frac{\pi}{2}\right)$

$$\cos\left(\frac{\pi}{2}\right) = x$$

$$\boxed{\cos\left(\frac{\pi}{2}\right) = 0}$$

b) $\tan\left(\frac{\pi}{2}\right)$

$$\tan\left(\frac{\pi}{2}\right) = \frac{y}{x}$$

$$\tan\left(\frac{\pi}{2}\right) = \frac{1}{0}$$

undefined!

c) $\sec(-270^\circ)$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{x}$$

$$\sec(-270) = \frac{1}{0}$$

undefined!

d) $\sin\left(-\frac{\pi}{2}\right)$

$$\sin\left(-\frac{\pi}{2}\right) = y$$

$$\boxed{\sin\left(-\frac{\pi}{2}\right) = -1}$$

e) $\cot\left(-\frac{\pi}{2}\right)$

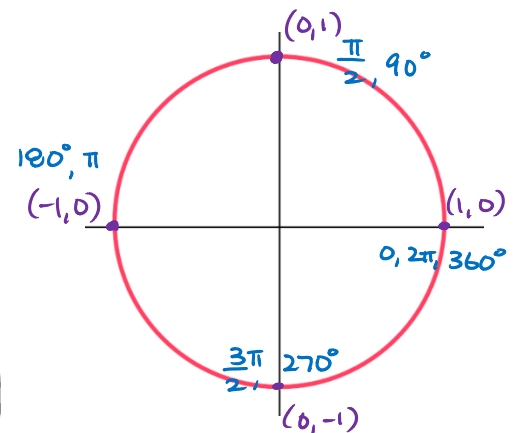
$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y}$$

$$\boxed{\cot\left(-\frac{\pi}{2}\right) = \frac{0}{1} = 0}$$

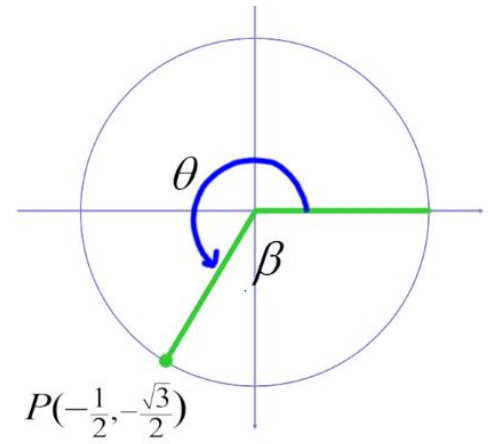
f) $\csc\left(-\frac{7\pi}{2}\right)$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{y}$$

$$\boxed{\csc\left(-\frac{7\pi}{2}\right) = \frac{1}{1} = 1}$$



- 2) Use the diagram to the right to:
- verify that P is on the unit circle.
 - find the six trig functions at θ .
 - compare to the 6 trig functions at β .



a) Show that the coord. of P satisfy $x^2 + y^2 = 1$

$$\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1$$

$$\frac{1}{4} + \frac{3}{4} = 1$$

$$1 = 1 \quad \checkmark$$

b) $\cos \theta = -\frac{1}{2}$ $\sin \theta = -\frac{\sqrt{3}}{2}$ $\tan \theta = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

$\sec \theta = -2$ $\csc \theta = \frac{2}{-\frac{\sqrt{3}}{2}} = -\frac{2\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ $\cot \theta = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c) They are the same!
Coterminal angles have the same trig values!

- 3) A point P is on the unit circle in quadrant III with y coordinate $-4/5$. Evaluate the six trig functions.

$P(x, -\frac{4}{5})$, must satisfy $x^2 + y^2 = 1$

$$x^2 + \left(-\frac{4}{5}\right)^2 = 1$$

$$x^2 + \frac{16}{25} = 1$$

$$x^2 = \frac{9}{25} \Rightarrow x = \pm \frac{3}{5}$$

Since P lies in quad III, $x = -\frac{3}{5}$.

$$P\left(-\frac{3}{5}, -\frac{4}{5}\right)$$

$\cos \theta = -\frac{3}{5}$ $\sec \theta = -\frac{5}{3}$

$\sin \theta = -\frac{4}{5}$ $\csc \theta = -\frac{5}{4}$

$\tan \theta = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$ $\cot \theta = \frac{3}{4}$

- 4) What does $\pi/2 < \theta < \pi$ mean?

θ lies in quad II

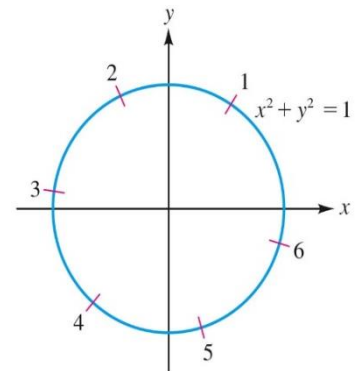


- 5) Estimating: Which is larger?

a) $\sin(3)$ or $\sin(1)$ $\sin(1)$

b) $\cos(1)$ or $\cos(2)$ $\cos(1)$

c) $\tan(1)$ or $\tan(4)$ $\tan(1)$



- 6) Use a calculator to evaluate $\sec(\theta)$, $\csc(\theta)$ and $\cot(\theta)$ for each given value of θ .

a) 30 $\sec 30 = 6.48$

$\csc 30 = -1.012$

$\cot 30 = -1.56$

b) -120° $\sec(-120^\circ) = -2$

$\csc(-120^\circ) = -1.15$

$\cot(-120^\circ) = 0.577$