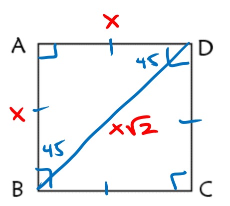
7.3
Notes

EVALUATING THE TRIGONOMETRIC FUNCTIONS

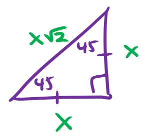
OBJECTIVES: 1) Derive the unit circle using special right triangles.
2) Complete a chart for the unit circle, from memory.
3) Use the unit circle to calculate trig functions of an angle.

DERIVING THE UNIT CIRCLE USING SPECIAL RIGHT TRIANGLES:

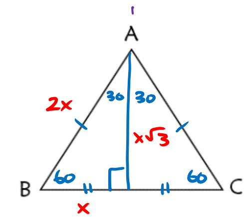
ABCD is a square.



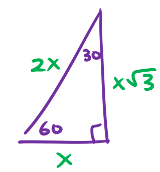
$x^2 + x^2 = c^2$
$2x^2 = c^2$
$c = \sqrt{2x^2}$
$c = x\sqrt{2}$

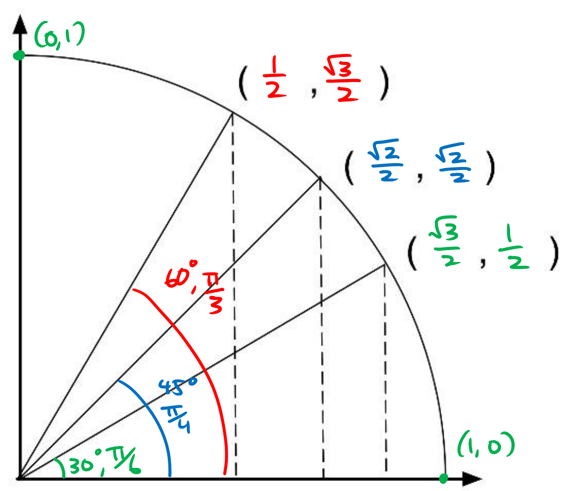


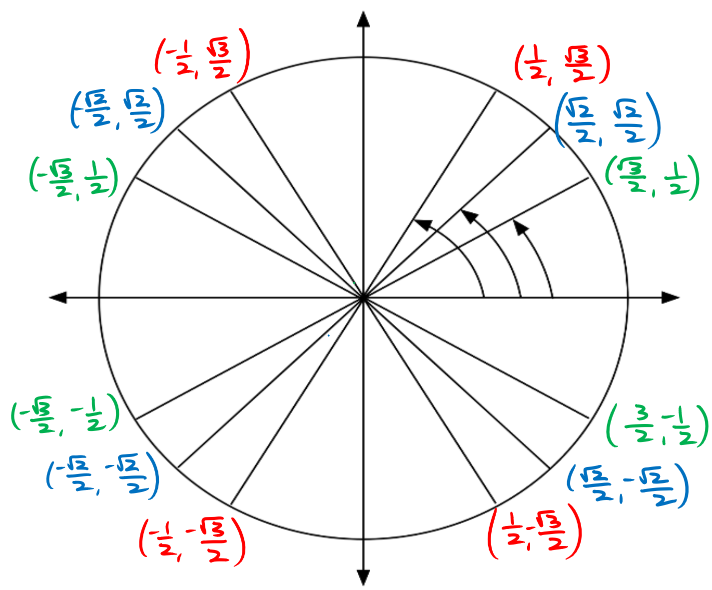
ΔABC is equilateral



$x^2 + b^2 = (2x)^2$
$b^2 = 4x^2 - x^2$
$b^2 = 3x^2$
$b = \sqrt{3x^2}$
$b = x\sqrt{3}$





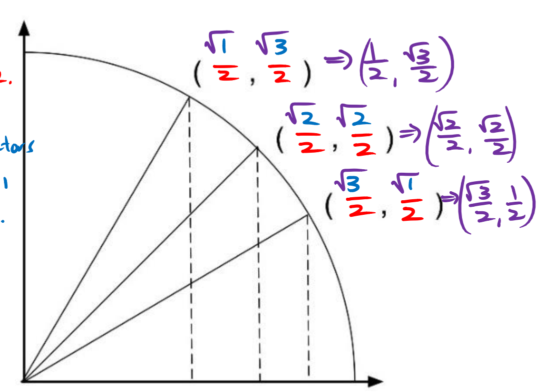


USING A MNEUMONIC TO MEMORIZE THE UNIT CIRCLE:

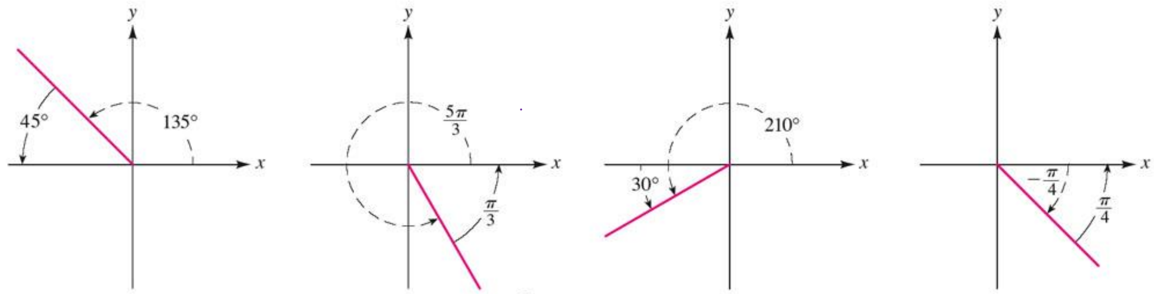
1) Write all fractions w/ denominator of 2.

2) Use 1,2,3 for numerators of x values. Use 3,2,1 for num. of y values.

3) Square root numerators & simplify.



REFERENCE ANGLES: The reference angle associated with θ is the acute angle (with positive measure) **formed by the x-axis (NOT the y-axis) and the terminal side of the angle.**



The reference angle for 135° is 45° [180° − 135° = 45°].

The reference number for $\frac{5\pi}{3}$ is $\frac{\pi}{3}$ [$2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$].

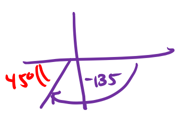
The reference angle for 210° is 30° [210° − 180° = 30°].

The reference number for $-\frac{\pi}{4}$ is $\frac{\pi}{4}$.

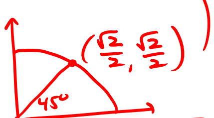
Figure 4

Examples:

a) $\sin(-135°)$

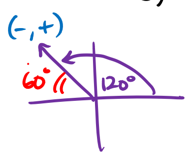


$\sin 45 = \frac{\sqrt{2}}{2}$

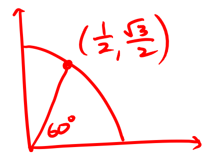


$\boxed{\sin(-135) = -\frac{\sqrt{2}}{2}}$

b) $\cot(120°)$



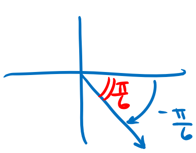
$\cot(60°) = \frac{1}{\tan\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



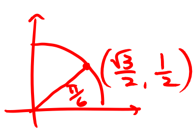
$\cot(60°) = \frac{\sqrt{3}}{3}$

$\boxed{\cot(120°) = -\frac{\sqrt{3}}{3}}$

c) $\sin\left(-\frac{\pi}{6}\right)$



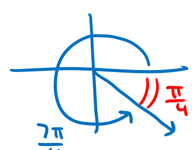
$\sin\left(\frac{\pi}{6}\right)$



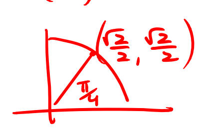
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\boxed{\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}}$

d) $\csc\left(\frac{7\pi}{4}\right)$



$\csc\left(\frac{\pi}{4}\right)$



$\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}}$

$\csc\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\boxed{\csc\left(\frac{7\pi}{4}\right) = -\sqrt{2}}$