7.4
Notes

ALGEBRA AND THE TRIG FUNCTIONS (PART 1)

OBJECTIVES: 1) Simplify and factor trig expressions.
2) Use trig identities to calculate trig functions.

NOTATION:

1) $\sin(\theta)$ is written as $\sin\theta$, except $\sin(A+B)$
2) $2(\sin\theta)(\cos\theta)$ written as $2\sin\theta\cos\theta$
3) $(\sin\theta)^2$ written as $\sin^2\theta$ except $(\sin\theta)^{-1}$

IDENTITIES:

1) $\sin^2\theta+\cos^2\theta=1$ 2) $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

$\sec\theta=\dfrac{1}{\cos\theta}$ $\csc\theta=\dfrac{1}{\sin\theta}$ $\cot\theta=\dfrac{1}{\tan\theta}$

1) Simplify: $2\cos^3\theta\sin^2\theta-7\sin^2\theta\cos^3\theta$

$2c^3s^2-7s^2c^3$

$-5c^3s^2$

$-5\cos^3\theta\sin^2\theta$

2) Factor: $3\cot^2\beta+\cot\beta-2$

$3c^2+c-2$

$(3c-2)(c+1)$

$(3\cot\beta-2)(\cot\beta+1)$

3) Simplify: $\dfrac{\csc A+1}{\cos A+\cot A}$

$\dfrac{\frac{1}{\sin A}+1}{\cos A+\frac{\cos A}{\sin A}}=\dfrac{\frac{\sin A+1}{\sin A}}{\frac{\cos A\sin A+\cos A}{\sin A}}$

$\dfrac{\sin A+1}{\sin A}\cdot\dfrac{\sin A}{\cos A\sin A+\cos A}=\dfrac{(\sin A+1)}{\cos A(\sin A+1)}$

$=\boxed{\dfrac{1}{\cos A}}=\sec A$

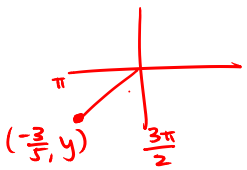
4) Simplify: $\dfrac{\sec A+1}{\sin A+\tan A}$

$\dfrac{\frac{1}{\cos A}+1}{\sin A+\frac{\sin A}{\cos A}}=\dfrac{\frac{1+\cos A}{\cos A}}{\frac{\sin A\cos A+\sin A}{\cos A}}$

$\dfrac{1+\cos A}{\cos A}\cdot\dfrac{\cos A}{\sin A\cos A+\sin A}=\dfrac{1+\cos A}{\sin A(\cos A+1)}$

$=\boxed{\dfrac{1}{\sin A}}=\csc A$

5) If $\cos\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find all 6 trig functions.



$x^2 + y^2 = 1$

$\left(\frac{-3}{5}\right)^2 + y^2 = 1$

$y^2 = \frac{16}{25}$

$y = \pm\frac{4}{5}$, since $\pi < \theta < \frac{3\pi}{2}$, $y = -\frac{4}{5}$

$P\left(\frac{-3}{5}, \frac{-4}{5}\right)$

$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\sin\theta = -\frac{4}{5}$ $\quad \csc\theta = -\frac{5}{4}$

$\tan\theta = \frac{-4/5}{-3/5} = -\frac{4}{5}\cdot\frac{-5}{3} = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

6) $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

$\frac{1+\sin\theta}{1-\sin^2\theta} + \frac{1-\sin\theta}{1-\sin^2\theta} = \frac{2}{1-\sin^2\theta} = \boxed{\frac{2}{\cos^2\theta}}$

$\cos^2+\sin^2\theta = 1$

$\cos^2\theta = 1-\sin^2\theta$

7) $\frac{1}{\sin t\cos t} - \frac{1}{\tan t}$

$= \frac{1}{\sin t\cos t} - \frac{\cos t}{\sin t}$

$= \frac{1}{\sin t\cos t} - \frac{\cos^2 t}{\sin t\cos t}$

$= \frac{1-\cos^2 t}{\sin t\cos t}$ $\quad \leftarrow \cos^2 t + \sin^2 t = 1$, $\sin^2 t = 1-\cos^2 t$

$= \frac{\sin^2 t}{\sin t\cos t}$

$= \frac{\sin t}{\cos t}$

$= \boxed{\tan t}$