7.4
Notes

ALGEBRA AND THE TRIG FUNCTIONS (PART 2)

OBJECTIVES: 1) Prove trig identities by expressing everything in terms of sine and cosine.
2) Create a difference of squares to prove a trig identity.

NOTATION:

1) $\sin(\theta)$ is written as $\sin\theta$, except $\sin(A+B)$
2) $2(\sin\theta)(\cos\theta)$ written as $2\sin\theta\cos\theta$
3) $(\sin\theta)^2$ written as $\sin^2\theta$ except $(\sin\theta)^{-1}$

IDENTITIES:

1) $\sin^2\theta+\cos^2\theta=1$ 2) $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

$\sec\theta=\dfrac{1}{\cos\theta}$ $\csc\theta=\dfrac{1}{\sin\theta}$ $\cot\theta=\dfrac{1}{\tan\theta}$

VERIFYING TRIG IDENTITIES:

1) They are not equations. You are not "solving" anything.
2) Work on one side of the equation at a time. (Typically pick the more complicated side).
3) Keep checking the side you aren't working on for progress.
4) Pencil only. You will make mistakes and go in circles at times.
5) Work up and down, using substitution and mathematically sound operations.
6) You have **VERIFIED THE IDENTITY** when both sides of the equal sign state the same thing.

STRATEGIES FOR VERIFYING IDENTITIES

1) Know, manipulate and substitute with the identities.
2) Change everything to sine and cosine.
3) Add and subtract fractions. Split fractions by sharing the denominator.
4) Factor expressions.
5) Get the same trig functions on each side of the equal sign.
6) Simplify as much as possible.
7) Multiply by the "conjugate".

EXAMPLES:

Verify each of the trig identities.

1) $\csc\alpha\tan\alpha=\sec\alpha$

CHANGE TO SIN/COS

$\frac{1}{\sin\alpha}\cdot\frac{\sin\alpha}{\cos\alpha}=$

$\frac{1}{\cos\alpha}=$

$\sec\alpha=$

2) $\cot\theta+1=\csc\theta(\cos\theta+\sin\theta)$

$=\frac{1}{\sin\theta}(\cos\theta+\sin\theta)$

$=\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}$

$=\cot\theta+1$

MULTIPLY BY THE CONJUGATE

3) $\dfrac{\cos\beta}{1-\sin\beta}=\dfrac{1+\sin\beta}{\cos\beta}$

$\dfrac{\cos\beta}{1-\sin\beta}\cdot\dfrac{1+\sin\beta}{1+\sin\beta}=$

$\dfrac{\cos\beta(1+\sin\beta)}{1-\sin^2\beta}=$

$\dfrac{\cos\beta(1+\sin\beta)}{\cos^2\beta}=$

$\dfrac{1+\sin\beta}{\cos\beta}=$

4) $\dfrac{\sin(\theta)-\cos(\theta)+1}{\sin(\theta)+\cos(\theta)-1}=\dfrac{\sin(\theta)+1}{\cos(\theta)}$

REALLY TOUGH ONE!

$\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}\cdot\dfrac{\sin\theta+\cos\theta+1}{\sin\theta+\cos\theta+1}=$

$\dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)+2\sin\theta+1}{(\sin\theta+\cos\theta)^2-1}=$

$\dfrac{\sin^2\theta-\cos^2\theta+2\sin\theta+1}{\sin^2\theta+2\cos\theta\sin\theta+\cos^2\theta-1}=\dfrac{2\sin^2\theta+2\sin\theta}{2\cos\theta\sin\theta}$

$\dfrac{2\sin\theta(\sin\theta+1)}{2\sin\theta\cos\theta}=\dfrac{\sin\theta+1}{\cos\theta}$

SPLIT THE FRACTION

5) $\dfrac{\tan A+\cot A}{\sec A\csc A}=1$

$\dfrac{\tan A}{\sec A\csc A}+\dfrac{\cot A}{\sec A\csc A}=$

$\dfrac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A}\cdot\frac{1}{\sin A}}+\dfrac{\frac{\cos A}{\sin A}}{\frac{1}{\cos A}\cdot\frac{1}{\sin A}}=$

$\dfrac{\sin A}{\cos A}\cdot\cos A\cdot\sin A+\dfrac{\cos A}{\sin A}\cdot\cos A\cdot\sin A$

$\sin^2A+\cos^2A=$

$1=$

6) $\dfrac{\tan x-\cot x}{\sin x\cos x}=\sec^2x-\csc^2x$

$\dfrac{\tan x}{\sin x\cos x}-\dfrac{\cot x}{\sin x\cos x}=$

$\dfrac{\frac{\sin x}{\cos x}}{\sin x\cos x}-\dfrac{\frac{\cos x}{\tan x}}{\sin x\cos x}=$

$\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\sin x\cos x}-\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin x\cos x}=$

$\dfrac{1}{\cos^2x}-\dfrac{1}{\sin^2x}=$

$\sec^2x-\csc^2x=$

COMBINE THE FRACTION

7) $\dfrac{1+\tan\beta}{1+\cot\beta}=\tan\beta$

$\dfrac{1+\frac{\sin\beta}{\cos\beta}}{1+\frac{\cos\beta}{\sin\beta}}=$

$\dfrac{\frac{\cos\beta+\sin\beta}{\cos\beta}}{\frac{\sin\beta+\cos\beta}{\sin\beta}}=$

$\dfrac{\cos\beta+\sin\beta}{\cos\beta}\cdot\dfrac{\sin\beta}{\sin\beta+\cos\beta}=$

$\dfrac{\sin\beta}{\cos\beta}=$

$\tan\beta=$

8) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=2\csc\theta$

$\dfrac{\sin^2\theta+(1+\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}=$

$\dfrac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}=$

$\dfrac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}=$

$\dfrac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}=$

$\dfrac{2}{\sin\theta}=$

$2\csc\theta=$