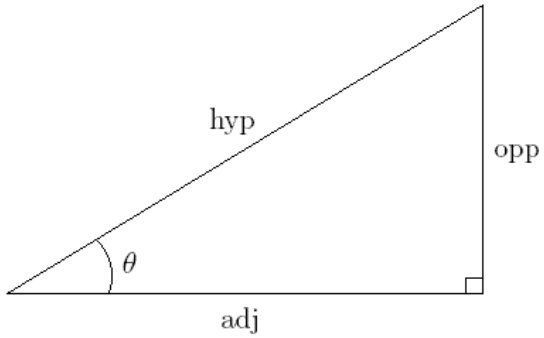
7.5
Notes

RIGHT TRIANGLE TRIG

OBJECTIVES: 1) Use right triangle trig definitions to extend trig functions beyond the unit circle.
2) Use the value of one trig function to find the value of all remaining trig functions.

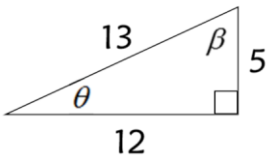
REVIEW RIGHT TRIANGLE TRIG DEFINITIONS:



$\sin\theta = \frac{opp.}{hyp.}$ $\quad \csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad \sec\theta = \frac{hyp}{adj}$

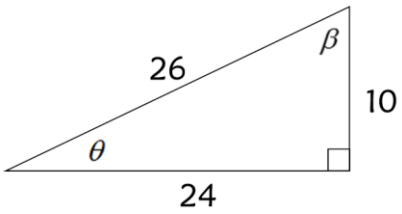
$\tan\theta = \frac{opp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$



$\sin\theta = \frac{5}{13}$ $\quad \sin\beta = \frac{12}{13}$

$\cos\theta = \frac{12}{13}$ $\quad \cos\beta = \frac{5}{13}$

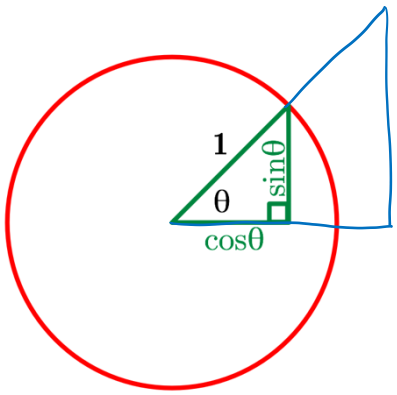
$\tan\theta = \frac{5}{12}$ $\quad \tan\beta = \frac{12}{5}$



$\sin\theta = \frac{10}{26} = \frac{5}{13}$ $\quad \sin\beta = \frac{12}{13}$

$\cos\theta = \frac{24}{26} = \frac{12}{13}$ $\quad \cos\beta = \frac{5}{13}$

$\tan\theta = \frac{10}{24} = \frac{5}{12}$ $\quad \tan\beta = \frac{12}{5}$



IDENTITIES:

$\sin(90° - \theta) = \cos(\theta)$ or $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

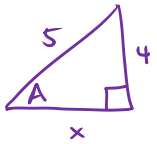
$\cos(90° - \theta) = \sin(\theta)$ or $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$

In words: If two acute angles are complementary (such as the two actue angles in a right triangle), then the sine of (either) one is the cosine of the other.

Cosine is shortened from phrase "complement's sine".

1) Given: $\sin A = \frac{4}{5}$. Find cosA and cotA if A is an acute angle.

$\sin^2 A + \cos^2 A = 1$



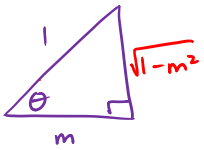
$x^2 + 4^2 = 5^2$

$x^2 = 9$

$\boxed{x = 3}$

$\boxed{\cos A = \frac{3}{5}}$ $\quad \cot A = \frac{1}{\tan A} = \frac{adj}{opp} = \boxed{\frac{3}{4}}$

2) Given: $\cos\theta = m$. Find all 6 trig functions in terms of m. Assume θ is an acute angle.



$1^2 = m^2 + b^2$

$1 - m^2 = b^2$

$\sqrt{1-m^2} = b$

$\cos\theta = m$ $\quad \sec\theta = \frac{1}{m}$

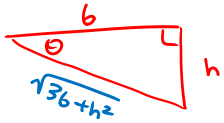
$\sin\theta = \sqrt{1-m^2}$

$\tan\theta = \frac{\sqrt{1-m^2}}{m}$ $\quad \csc\theta = \frac{1}{\sqrt{1-m^2}} = \frac{\sqrt{1-m^2}}{1-m^2}$

$\cot\theta = \frac{m}{\sqrt{1-m^2}} = \frac{m\sqrt{1-m^2}}{1-m^2}$

3) Find all six trig functions if $\tan\theta = -\frac{h}{6}$ and $\frac{3\pi}{2} < \theta < 2\pi$.

$\tan\theta = \frac{-h}{6}$



$6^2 + h^2 = c^2$

$c = \sqrt{36+h^2}$

$\sin\theta = \frac{-h}{\sqrt{36+h^2}}$ $\quad \csc\theta = \frac{\sqrt{36+h^2}}{-h}$

$\cos\theta = \frac{6}{\sqrt{36+h^2}}$ $\quad \sec\theta = \frac{\sqrt{36+h^2}}{6}$

$\tan\theta = \frac{-h}{6}$ $\quad \cot\theta = \frac{-6}{h}$